Incidence and Growth of Patent Thickets - The Impact of Technological Opportunities and Complexity

Georg von Graevenitz[*], Stefan Wagner[†], Dietmar Harhoff[‡]

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Abstract

We investigate incidence and evolution of patent thickets. Our empirical analysis is based on a theoretical model of patenting in complex and discrete technologies. The model captures how competition for patent portfolios and complementarity of patents affect patenting incentives. We show that lower technological opportunities increase patenting incentives in complex technologies while they decrease incentives in discrete technologies. Also, more competitors increase patenting incentives in complex technologies and reduce them in discrete technologies. To test these predictions a new measure of the density of patent thickets is introduced. European patent citations are used to construct measures of fragmentation and technological opportunity. Our empirical analysis is based on a panel capturing patenting behavior of 2074 firms in 30 technology areas over 15 years. GMM estimation results confirm the predictions of our theoretical model. The results show that patent thickets exist in 9 out of 30 technology areas. We find that decreased technological opportunities are a surprisingly strong driver of patent thicket growth.
JEL: L13, L20, O34.

Keywords: Patenting, Patent thickets, Patent portfolio races, Complexity, Technological Opportunities.

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[*] Georg von Graevenitz, Ludwig Maximilians University, Munich School of Management, INNO-tec, Kaulbachstraße 45, D-80539, Munich, graevenitz@lmu.de

[†] Stefan Wagner, Ludwig Maximilians University, Munich School of Management, INNO-tec, Kaulbachstraße 45, D-80539, Munich, swagner@bwl.lmu.de

[‡] Dietmar Harhoff, Ludwig Maximilians University, Munich School of Management, INNO-tec, Kaulbachstraße 45, D-80539, Munich, harhoff@lmu.de

1 Introduction

Strong increases in the level of patent applications have been observed at the United States Patent and Trademark Office (USPTO) (Kortum and Lerner, 1998, Hall, 2005) as well as the European Patent Office (EPO) (von Graevenitz et al., 2007). These "patent explosions" pose serious challenges for existing patent systems and also for competition authorities.[1]

Explanations for the shift in patenting behavior focus on changes in the legal environment and management practices, the complexity of technologies, greater technological opportunities and increased strategic behavior on the part of firms. While it has been shown that most of these factors play a role empirically, there are no explanations of patenting behavior that model the joint effects of these determinants.[2] We investigate empirically how complexity and technological opportunity interact to determine firms' patenting choices using data on patenting in Europe. Our main innovation consists of a new measure of complexity of blocking relationships which allows us to quantify the extent of patent thickets. We also provide a theoretical model which characterizes how technological opportunity and complexity of technologies interact in determining firms' patenting efforts. We estimate a reduced form empirical specification which provides support for the predictions we derive theoretically. Our results indicate that patenting responds surprisingly strongly to variation in technological opportunities. We find that patent thickets exist in 9 of the 30 technology areas. Finally, descriptive analysis shows that patent thickets are becoming more complex and widespread over time.

Kortum and Lerner (1998) investigated the explosion of patenting at the USPTO which began around 1984 (Hall, 2005). By a process of elimination Kortum and Lerner (1998, 1999) argue that the shift towards increased patenting is mainly the result of changed management practices making R&D more applied and raising the yield of patents from R&D. In contrast, Hall and Ziedonis (2001) argue that the patenting surge is a strategic response to an increased threat of hold-up in complex technologies. This threat resulted from the "pro-patent" legal environment ushered in after the establishment of the Court of Appeals for the Federal Circuit in the United States (Jaffe, 2000). In this changed environment hold-up ensues if blocking patents are enforced through the courts or if high settlement payments can be extracted in the presence of high costs for court proceedings. Complexity of a technology implies that patents are naturally complements, and therefore hold-up is likely to arise in the process of negotiations over licenses if firms enforce their patents (Shapiro, 2001, 2006). Kortum and Lerner (1998, 1999) and Hall and Ziedonis (2001) explore whether enhanced fertility of R&D led to an increase in patent filings, but do not find systematic evidence for such an influence.

The R&D and patenting literature has made extensive use of the distinction between dis-

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[1] For extensive discussions of the policy questions surrounding current functioning of the patent systems in the United States and in Europe refer to National Research Council (2004), F.T.C. (2003), Jaffe and Lerner (2004), von Graevenitz et al. (2007) and Bessen and Meurer (2008).

[2] Formal models of patenting abound, for a survey of this literature refer to Scotchmer (2005) or Gallini and Scotchmer (2002). Formal models of patenting in patent thickets do not attempt to span both complex and discrete technologies as we do here: Bessen (2004),Clark and Konrad (2005) and Siebert and von Graevenitz (2006). These models build on the patent race literature pioneered by Loury (1979), Lee and Wilde (1980), Reinganum (1989) and Beath et al. (1989).

crete and complex technologies. In the former group of technologies, one (or very few) patents suffice to protect a product or process, while in complex technologies, products and processes require a multitude of complementary IP rights in order to be commercialized without infringement. Our model of patenting covers both complex and discrete technologies. It captures competition for patent portfolios and efficiency gains from additional patents in complex technologies. Technological opportunities, complexity of a technology and patenting costs jointly determine patenting levels. We model the choice between pursuing new technological opportunities and deepened protection of existing technologies by patenting "facets" of technological opportunities. We show firms patent *less* in response to increasing technological opportunities in complex technologies and *more* if more other firms compete for patents. Both effects result from strategic interaction of firms in a complex technology: greater technological opportunities reduce pressures on firms to defend their stake in existing technologies by patenting heavily, whereas greater competition increases this pressure. In contrast to previous studies, our model shows that less technological opportunities enhance the intensity with which each individual opportunity is pursued, leading *ceteris paribus* to more patenting.

We test the model using a comprehensive dataset based on EPO patent data. It comprises information on patenting behavior between 1987 and 2003, covering all patentable technologies. This allows us to identify differences in patenting behavior between complex and discrete technologies. We construct a novel measure of the complexity of blocking in a technology based on information specific to European patents. Our measure exploits the fact that patent examiners at the EPO indicate which prior patents block or restrict the breadth of the patent application under review. We count how often three or more firms apply for mutually blocking patents within a three year period. This gives rise to a count of mutually blocking firm *Triples*. The measure captures effects of complex blocking relationships which arise in technologies even if patent ownership remains relatively concentrated. We validate this new measure by showing that greater incidence of complex blocking relationships is correlated with existing measures of technological complexity, such as the one suggested by Cohen et al. (2000).

Additionally, a measure of technological opportunity is needed to test our hypotheses. We use the extent to which patents reference non-patent literature for this purpose. Meyer (2000), Narin and Noma (1985) and Narin et al. (1997) show that the share of references pointing to non-patent literature (which consists mostly of scientific publications) is a good proxy for the strength of the science link of a technology. The strength of the science link within a technology area will indicate how much technological opportunity there is at a given time.

Patenting behavior is known to be highly persistent, due to the long term nature of firms' R&D investment decisions. We control for the persistence of patenting which arises from long term R&D investment decisions by including a lagged dependent variable in the empirical model. The model is estimated using systems GMM estimators (Blundell and Bond, 1998, Arellano, 2003, Alvarez and Arellano, 2003) to control for endogeneity of the lagged dependent variable. Additionally, we treat our measures of technological opportunity, complexity and fragmentation as predetermined. Evidence from GMM regressions as well as results from OLS and a fixed effects estimator support the theoretical predictions we derive from our model. In particular, we find that decreasing technological opportunities in conjunction with increasing complexity lead to more patent filings. Thus, our paper suggests a new rationale why patent filings may have risen since the mid-1980s.

The remainder of this paper is structured as follows. Section 2 provides a theoretical model of patenting which explains firms' patenting strategies. We derive three hypotheses from this model that are empirically testable. In Section 3 we describe our dataset and the variables we employ to analyze firms' patenting behavior. As there is little cross industry evidence of patenting trends at the EPO, Section 4 provides a descriptive analysis of these trends, focusing particularly on our own measure and alternative measures of complexity. Section 5 provides the empirical model and results and Section 6 concludes.

2 A Model of Patenting

In this section we present a model of patenting behavior. This disentangles the influences of technological opportunity and of complexity of blocking relationships on patenting. Both opportunities and complexity are assumed to be fixed in the short- to medium term.[3] First, we motivate the model. Then, we discuss assumptions and solve the model, presenting several predictions. These underpin the empirical results presented in Sections 4 and 5 below.

2.1 Structure of the Model

We focus on patenting within technology areas. Each area is characterized by a number of technological opportunities representing independent sources of profit to firms. Theses profits are appropriated by patenting "facets" of the opportunity. Each facet represents a separate patentable innovation that contributes to the exploitation of a technological opportunity. The complexity of a technology area is higher if there are many facets within technological opportunities. Then, more different firms may own facets on technological opportunities and the share of profits appropriated with patents on individual facets decreases. This raises pressure to patent giving rise to one patenting incentive in a complex technology.

In complex facets on a technological opportunity protect complementary technological solutions. As more facets are patented the total value of patents on the technological opportunity rises. This gives rise to a second patenting incentive.

The number of patents available in a technology area is determined by the extent of technological opportunities and their facets. For instance, a technological opportunity might become available through university research. A firm may use such publicly available knowledge for the private development of a certain chemical compound in organic chemistry or for the search for a drug in pharmaceuticals. Complexity arises if it is possible to patent several facets within

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[3] In the long run technological opportunity may be affected by firms' patenting efforts. Unravelling this question will require a separate study with data including information on firms' R&D investments over a long period.

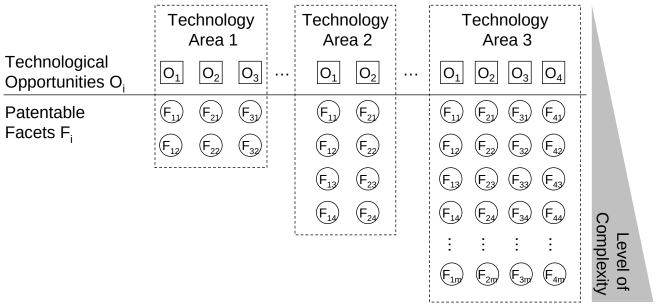

Figure 1: Relation between complexity and the number of patentable facets per technological opportunity. The picture demonstrates how the number of patents grows as complexity and the number of patentable facets increase.

technological opportunities. More patentable facets increase the number of firms who might own patents relating to the same technological opportunity. The resulting threat of hold-up induces firms to hoard patents. Where opportunities consist of one facet only, technology is discrete. Here firms may anticipate competition for patents, but also know that each patent has a value independently of others' patents. Figure 1 presents this idea.

The total set of patentable inventions in a technology (Ω) consists of O technology opportunities and F facets such that: $FO = \Omega$. Variation in the two dimensions of this set arises for different reasons. Changes in the number of technological opportunities available at a specific time affect O. Current efforts in basic R&D open additional new opportunities in the future. In contrast, the number of facets which are patentable on a given opportunity depends mainly on institutional and legal factors. Most importantly the breadth of patents granted by a patent office will determine how many facets are patentable. The broader each patent the fewer facets will be available on each technological opportunity. As an example, consider the well-known strength of patent protection in pharmaceutical technology. Patent laws allow applicants to receive broad patent protection for a variety of similar molecules by filing one patent application. As a consequence, one patent suffices for the applicant to protect a drug effectively against attempts to invent around the patent. Conversely, in consumer electronics, one commercial product such as a TV set, or a production process in semiconductor manufacturing are protected through hundreds of patent rights. Leaving aside the impact of legally defined patent breath, the ability of a patent office to prevent overlap of patents affects the number of facets available. If a patent office has few resources to check patent applications carefully it is likely that many granted patents overlap. Where firms anticipate this, the effective breadth of each patent application is reduced and more facets become available for patenting.

2.2 Assumptions and Definitions

A technology area is characterized by the technological opportunities it offers (O) and the facets which firms can patent per technological opportunity (F). We assume that each technological opportunity offers the same number of facets. The number of facets determines the complexity of a technology area. A technology area is discrete if $F = 1$. Each technological opportunity is associated with a maximal total value $V(F)$ and an attained maximal value $V(\tilde{F})$. The attained value depends on the number of facets actually patented $\tilde{F}$, which may be less or equal to the number of available facets F. Firms appropriate a share of the attained value by acquiring patents. The levels of O, F and V are public knowledge.

The model incorporates two dimensions of choice for each firm i, the number of opportunities each firm invests in (o_i) and the number of facets on each opportunity which the firm seeks to patent (f_i). Note that each firm can only make one patent application per facet $(f_i \in [0, F])$ and that firms can only patent in technological opportunities which they have researched $(o_i \in [0, O])$. Firms face a trade-off between patenting more facets per opportunity and patenting in more different technological opportunities. While patenting additional facets is assumed to be costless,[4] firms must undertake additional R&D on each technology opportunity they turn to. Additionally, they must pay maintenance fees on granted patents.

Firms choose simultaneously how many technological opportunities to invest in and how many facets per opportunity to try to patent. Strategic interaction arises as the probability that a patent application is granted depends on the level of rivals' patent applications on a given facet. We assume that the patent office will grant each application for a patent on a facet with equal probability, but only grants one patent overall on the facet.

Then for each technological opportunity, the expected number of facets that become granted patents is given by the following expression, where we define $\phi_{ko} = f_{ko}/F$:

$$\tilde{F}_o = F\left[1 - (1 - \phi_{io}) \prod_{j=1}^{N_O - 1} (1 - \phi_{jo})\right] \quad , \tag{$\tilde{F}$}$$

Note that the subscripts $_{i,j}$ denote the firm and $_o$ denotes a given technological opportunity.

The number of patented facets increases in the number of firms investing in the technology opportunity N_O and in the the share of facets each firm seeks to patent ϕ_{ko}. The expressions shows that the total share of facets covered by at least one applicant is one minus the share of facets that attract no applications at all. Each covered facet will also be patented. We derive an expression for N_O in Appendix A.1. In Appendix A.2 we show that the number of facets covered increases in the complexity of the technology, in the number of rivals investing in a technological opportunity and also in the number of opportunities each firm invests in:

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[4] We make this assumption in order to simplify the model, but it can be shown that it does not affect our results if patent filing costs are sufficiently low in comparison to the costs of maintenance. In practice, initial application and examination fees for patents are indeed much lower than post-grant translation and renewal fees, since most patent offices cross-subsidize the initial stages in order to encourage patent filing.

$$\frac{\partial \tilde{F}_o}{\partial F} > 0 \quad , \quad \frac{\partial \tilde{F}_o}{\partial N_{Oo}} > 0 \quad \text{and} \quad \frac{\partial \tilde{F}}{\partial o_{jo}} > 0 \quad . \tag{1}$$

The first effect arises as more facets can be patented as technology becomes more complex.

We assume that the value of the technology increases in the number of facets patented $\tilde{F}$:

$$\frac{\partial V_o}{\partial \tilde{F}_o} > 0 \qquad\qquad . \tag{V}$$

Now turn to the probability that a firm patents a given facet. This depends on the expected number of rivals for the facet and the probability that each possible number of rivals arises. In Appendix A.3 we show that the probability of patenting a facet can be expressed as:

$$p_{ko} = \sum_{i=0}^{N_O} \frac{1}{i+1} \binom{N_O}{i} \prod_{l=0}^{N_O-i} (1-\phi_{lo}) \prod_{l=N_O-i}^{N_O} \phi_{lo} \quad . \tag{p}$$

This expression shows that the probability of obtaining a patent on an application is a sum of weighted probabilities. Each element of the sum consists of the weighted probability of obtaining a patent given the number of rival firms also seeking a patent on the facet $1/(1+i)$. The weight expresses the probability of observing a given number of rivals. In Appendix A.3 we show that the probability of patenting a facet decreases in the level of facets rival firms seek to patent and in the number of rival firms investing in a given technological opportunity:

$$\frac{\partial p_{ko}}{\partial \phi_{jo}} < 0 \; , \quad \frac{\partial p_{ko}}{\partial N_{Oo}} < 0 \quad \text{and} \quad \frac{\partial p_{ko}}{\partial o_{jo}} < 0 \; . \tag{2}$$

As the number of facets per opportunity grows, so does the probability that different firms own patents belonging to an opportunity. Hold-up becomes increasingly likely. Then, firms need to disentangle ownership rights, giving rise to legal costs (L). We do not explicitly model this process. The literature on patent thickets and complex technology shows that several institutional arrangements allow firms to disentangle overlapping property rights - these include licensing, patent pools, standard setting as well as litigation (Shapiro, 2001). Irrespective of the precise mechanism used to prevent or resolve hold-up, the patenting explosion is driven primarily by the prevailing assumption of patenting firms that those with a larger share of patents on a technology benefit substantially in reducing the costs of hold-up (Grindley and Teece, 1997, Shapiro, 2001, Ziedonis, 2004). Additional patents reduce marginal legal costs as the share of patents grows, since firms with a large share of patents on a technological opportunity will need to cross-license or litigate less. Therefore, we assume:

$$\frac{\partial L}{\partial s_{io}} > 0 \quad , \quad \frac{\partial^2 L}{\partial {s_{io}}^2} < 0 \qquad , \tag{L}$$

where s_{ij} is the share of granted patents firm i obtains in technological opportunity o.

Three additional sources of patenting costs are recognized in our model:

i Per opportunity a firm invests in, it faces a fixed cost of R&D: C_o.

ii Per granted patent a firm faces costs of administering and enforcing that patent: C_a.

iii The coordination of R&D on *different* technological opportunities imposes costs $C_c(o_{io})$. Therefore, we assume that $\frac{\partial C_c}{\partial o_{io}} > 0$.

2.3 Solving the Model

The expected value of patenting for firm i in a technology area is:

$$\pi_i(o_i, \boldsymbol{f}_i) = \left(\sum_{m=1}^{o_i} \Big[V(\tilde{F}_o) s_{io} - L(s_{io}) \Big] - C_o - s_{io} \tilde{F} C_a \right) - C_c(o_i) \quad , \tag{3}$$

where $\boldsymbol{f}_i$ is a $O \times 1$ vector containing the number of facets for each technological opportunity which firm i seeks to patent. The expression shows that firms' profits consist of revenues derived per technological opportunity and costs of coordinating R&D across different technological opportunities. Revenues per technological opportunity depend on the share of value obtained, legal costs as well as costs of R&D on the technological opportunity and costs of administering granted patents.

Given this objective function, we characterize the game firms are playing:

- There are $N + 1$ firms.
- Each firm simultaneously chooses the number of technological opportunities $o_i \in [0, O]$ and the vector of facets $\boldsymbol{f}_i$ containing the number of patents applied for per opportunity $f_{io} \in [0, F]$, to maximize the payoff function π_i. Firms' strategy sets S_n are elements of $R^{(O+1)}$.
- Firms' payoff functions π_i, defined in equation (3), are twice continuously differentiable and depend only on rivals' aggregate strategies.

Firms' payoffs depend on their rivals' aggregate strategies because the probability of obtaining a patent on a given facet is a function of rivals' patent applications.

First notice that this game is symmetric as it is exchangeable in permutations of the players. This implies that symmetric equilibria exist if the game can be shown to be supermodular (Vives, 2005).[5] Next, notice that technological opportunities within a technology area are symmetrical in our model: they have the same number of facets and costs of R&D are the same. There are no attributes of individual technological opportunities that distinguish these from others. From the perspective of rival firms we can represent each firm's choice of the

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[5] Note also that only symmetric equilibira exist as the strategy spaces of players are completely ordered.

vector of facets to patent ($\boldsymbol{f}_i$) by the average number of facets ($\tilde{f}_i$) to patent across all opportunities. This average represents the expected number of facets the firm will patent per opportunity. Nothing else matters for rival firms as there is no scheme which will allow firms to coordinate applications on specific technological opportunities in our model. Then, we can represent each firm's strategy as a function of the number of technological opportunities invested in and the average number of facets chosen per opportunity. Firms' objective functions may be rewritten as follows:

$$\max_{o_i,\tilde{f}_i} \tilde{\pi}_i(o_i,\tilde{f}_i) = o_i\left(\left[V(\tilde{F})s_i - L(s_i)\right] - C_o - \tilde{f}_i p_i C_a\right) - C_c(o_i) \quad , \tag{4}$$

where $s_i = \tilde{f}_i p_i / \tilde{F}$ are the symmetric expected shares of granted patents per technological opportunity. Given this definition of firms' objective functions, we can show that:

Proposition 1
The game in which firms maximize $\tilde{\pi}$ choosing o_i and $\tilde{f}_i$ is smooth supermodular if the value of technology is not excessively concave in granted patents.

This implies, that the game will not be smooth supermodular if the technology is not complex. By definition in that case there is only one facet $(F = 1)$ per technological opportunity. We characterize this case further below.

To prove Proposition 1 we show that firms' profit functions are supermodular (i) in their own actions and (ii) in every combination of their own actions with those of rival firms (Milgrom and Roberts, 1990, Vives, 1999, Amir, 2005). This is the case if the cross-partial derivatives between own as well as own and rival actions are positive.

To begin with we derive the first order conditions:

$$\frac{\partial \tilde{\pi}}{\partial o_i} = V s_i - L(s_i) - C_o - \tilde{f}_i p_i C_a - \frac{\partial C_c}{\partial o_i} = 0 \quad , \tag{5}$$

$$\begin{aligned} \frac{\partial \tilde{\pi}}{\partial \tilde{f}_i} &= o_i \left(V \frac{p_i}{\tilde{F}} - \frac{\partial L}{\partial s_i} \frac{p_i}{\tilde{F}} - p_i C_a + \frac{s_i}{\tilde{F}} \left[\frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_i} \right] \frac{\partial \tilde{F}}{\partial \tilde{f}_i} \right) \\ &= \frac{o_i p_i}{\tilde{F}} \left(\left[V - \frac{\partial L}{\partial s_i} \right] \left(1 - \epsilon_{\tilde{F}\tilde{f}_i}\right) - \tilde{F} C_a + \frac{\partial V}{\partial \tilde{F}} \tilde{F} \epsilon_{\tilde{F}\tilde{f}_i} \right) = 0 \quad . \end{aligned} \tag{6}$$

These constitute a system of implicit relations which determine the optimal choice of opportunities ($\hat{o}_i$) and facets ($\hat{f}_i$) chosen by each firm in equilibrium.

Equation (5) shows that given $\tilde{f}_i$ investment in an additional technological opportunity raises firms' expected revenues by the share of that opportunity's value which they expect to obtain. It also raises their costs as additional fixed costs of R&D, additional legal and administrative costs as well as coordination costs arise.

Equation (6) shows that given o_i, patent applications on additional facets will affect profits in two ways: they change the expected number of facets the firm will own and they change the number of facets likely to be patented overall. The first, direct effect raises revenue but also costs. The second effect is indirect: an increase in the number of facets applied for raises the expected number of facets patented. This makes the technological opportunity more valuable, and simultaneously dilutes the value of a given share of facets owned on the technology. Overall, this indirect effect is positive as long as the value of the technological opportunity is not too concave in the number of granted patents $\tilde{F}$.

A corner solution for f_i can be ruled out if the value of the technology (V) is concave in the number of facets covered $(\tilde{F})$. In Appendix A.2 we show that $1 \geq \epsilon_{\tilde{F}\tilde{f}_i} \geq 0$ and that the elasticity goes to one if $\tilde{f}_i = F$. Concavity of V implies firms will not cover all possible facets if $\frac{\partial V}{\partial \tilde{F}} < C_a$ for $\tilde{F} = F$, but will cover some facets for $\tilde{F} < F$ if $\frac{\partial V}{\partial \tilde{F}} > C_a$. If the marginal gain from a patent $(\frac{\partial V}{\partial \tilde{F}})$ is greater than its marginal administrative cost (C_a) when few patents have been granted on a technological opportunity this restriction applies.

Next we show when firms' profit functions are supermodular, given first order conditions (5) and (6). First, we derive the cross partial derivative with respect to firms' own actions:

$$\frac{\partial^2 \tilde{\pi}_i}{\partial o_i \partial \tilde{f}_i} = \left(V \frac{p_i}{\tilde{F}} - \frac{\partial L}{\partial s_i}\frac{p_i}{\tilde{F}} - p_i C_a + \frac{s_i}{\tilde{F}}\left[\frac{\partial V}{\partial \tilde{F}}\tilde{F} - V + \frac{\partial L}{\partial s_i}\right]\frac{\partial \tilde{F}}{\partial \tilde{f}_i}\right) = 0 \tag{7}$$

This expression corresponds to the first order condition (6) for the optimal number of facets. Now consider effects of rivals' actions on firms' own actions:

$$\frac{\partial^2 \tilde{\pi}_i}{\partial o_i \partial o_j} = \frac{\tilde{f}_i}{\tilde{F}}\left[\frac{p_i}{\tilde{F}}\left(\frac{\partial V}{\partial \tilde{F}}\tilde{F} - V + \frac{\partial L}{\partial s_i}\right)\frac{\partial \tilde{F}}{\partial o_j} + \left[V - \frac{\partial L}{\partial s_i} - \tilde{F}C_a\right]\frac{\partial p_i}{\partial o_j}\right], \tag{8}$$

$$\frac{\partial^2 \tilde{\pi}_i}{\partial o_i \partial \tilde{f}_j} = \frac{\tilde{f}_i}{\tilde{F}}\left[\frac{p_i}{\tilde{F}}\left(\frac{\partial V}{\partial \tilde{F}}\tilde{F} - V + \frac{\partial L}{\partial s_i}\right)\frac{\partial \tilde{F}}{\partial f_j} + \left[V - \frac{\partial L}{\partial s_i} - \tilde{F}C_a\right]\frac{\partial p_i}{\partial f_j}\right], \tag{9}$$

$$\begin{aligned}\frac{\partial^2 \tilde{\pi}_i}{\partial \tilde{f}_i \partial o_j} = {} & \left[\frac{\partial V}{\partial \tilde{F}} + \frac{\partial^2 V}{\partial \tilde{F}^2}\tilde{F}\epsilon_{\tilde{F}\tilde{f}_i} - C_a\right]\frac{\partial \tilde{F}}{\partial o_j} + \left(\frac{\partial V}{\partial \tilde{F}}\tilde{F} - V + \frac{\partial L}{\partial s_i}\right)\frac{\partial \epsilon_{\tilde{F}\tilde{f}_i}}{\partial o_j} \\ & + \left[\frac{\partial \tilde{F}}{\partial o_j}\frac{p_i}{\tilde{F}} - \frac{\partial p_i}{\partial o_j}\right]\frac{\partial^2 L}{\partial {s_i}^2}\frac{\tilde{f}_i}{\tilde{F}}\left(1 - \epsilon_{\tilde{F}\tilde{f}_i}\right) \quad ,\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{\partial^2 \tilde{\pi}_i}{\partial \tilde{f}_i \partial \tilde{f}_j} = {} & \left[\frac{\partial V}{\partial \tilde{F}} + \frac{\partial^2 V}{\partial \tilde{F}^2}\tilde{F}\epsilon_{\tilde{F}\tilde{f}_i} - C_a\right]\frac{\partial \tilde{F}}{\partial f_j} + \left(\frac{\partial V}{\partial \tilde{F}}\tilde{F} - V + \frac{\partial L}{\partial s_i}\right)\frac{\partial \epsilon_{\tilde{F}\tilde{f}_i}}{\partial f_j} \\ & + \left[\frac{\partial \tilde{F}}{\partial f_j}\frac{p_i}{\tilde{F}} - \frac{\partial p_i}{\partial f_j}\right]\frac{\partial^2 L}{\partial {s_i}^2}\frac{\tilde{f}_i}{\tilde{F}}\left(1 - \epsilon_{\tilde{F}\tilde{f}_i}\right) \quad .\end{aligned} \tag{11}$$

Consider equations (8) and (9). These are positive if the following inequalities are satisfied:

$$V - \frac{\partial V}{\partial \tilde{F}}\tilde{F} < \frac{\partial L}{\partial s_i} \qquad\qquad V - \frac{\partial L}{\partial s_i} < \tilde{F}C_a. \tag{12}$$

Using Equation (6) it can be shown that the inequality on the right is $-\epsilon_{\tilde{F}\tilde{f}_i}$ times the inequality on the left. Both inequalities are satisfied if the value of the technology is either convex or slightly concave in the level of granted patents $(\tilde{F})$. Equations (10) and (11) are also positive as long as the value of the technology is not too concave in the level of granted patents. If the technology is convex there is a corner solution in which $f_i = F$. Then, $\tilde{F} = F$ and $\tilde{F}$ no longer varies with f_j or o_j. Similarly $\epsilon_{\tilde{F}\tilde{f}_i} = 1$. The corner solution means that equations (7) and (8) are positive and Equations (9)- (11) are zero.

For example assume that: $V(\tilde{F}) = K\tilde{F}^\alpha$, where $K > C_a$. This function is strictly concave in granted patents for $1 > \alpha > 0$ and convex in granted patents for $\alpha \geq 1$. Condition (12) holds if α is less than but sufficiently close to one or if $\alpha > 1$: $\frac{\partial L}{\partial s_i} > K\tilde{F}^\alpha(1-\alpha)$. Turning to the first terms in square brackets in Equations (10) and (11) concavity and $K > C_a$ jointly imply $K\alpha^2 F^{\alpha-1} > C_a$. As α becomes close to one, the terms in square brackets are positive.[6]

Finally, note that the last term in equations (10) and (11) is always negative by Assumption (L). However, this term will be negligible for two reasons: if legal costs are only slightly concave in the share of patents obtained then $\frac{\partial^2 L}{\partial s_i^2}$ is small. Additionally, the product $\frac{\tilde{f}_i}{\tilde{F}}\left(1-\epsilon_{\tilde{F}\tilde{f}_i}\right)$ will be small whenever a firm has covered a large or a small share of all available facets.

Firms' own and rivals' patenting efforts are strategic complements if technology is complex in the game analyzed here. Rival firms patenting efforts raise the value of each technological opportunity that these rivals invest in. This raises incentives for firms to invest in facets and technological opportunities themselves. Additionally, firms facing more investments by rivals will seek to counter these in order to prevent dilution of their share of granted patents in a technology. In particular, the game is only smooth supermodular if legal costs do not decline severely as firms' share of patents on a technological opportunity increase and if the value of a technological opportunity is not too concave in the number of patents granted. Both conditions guarantee that further patenting by rivals raises the value of patenting to each firm.

Patent rights on technological opportunities often overlap (Lemley and Shapiro, 2005). Then, it is likely that additional patents add less and less to the overall value of a technological opportunity and the requirement that the total value of a technological opportunity is concave in all patents granted is natural. However, our analysis also shows that the game analyzed is smooth supermodular even if there is a corner solution in which $\tilde{f}_i = \tilde{F} = F$ as the set from which f_i is chosen is increasing in F. Next, consider legal costs. We noted previously that firms build patent portfolios in order to ensure against hold-up and to increase bargaining power in negotiations (Grindley and Teece, 1997, Shapiro, 2001, Hall and Ziedonis, 2001, Lemley, 2001). While greater portfolios may dampen the marginal increase of legal costs associated with additional granted patents, this effect is likely to be small. Each patent is probabilistic (Lemley and Shapiro, 2005) and adding it to a portfolio therefore adds the possibility of a legal challenge, raising costs. Therefore, overall we regard the assumptions guaranteeing supermodularity of this game to be defensible.

Comparative Statics of the Model

Now consider comparative statics if a technology is complex $(F > 1)$ and Proposition 1 holds. First, we derive a Corollary to Proposition 1:

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[6] Note that here we have assumed that $\epsilon_{\tilde{F}\tilde{f}_i} = 1$ which makes it harder to show that concavity suffices to make the expression positive. We show in Appendix A.2 that $1 \geq \epsilon_{\tilde{F}\tilde{f}_i} \geq 0$.

Corollary 1

An increase in the number of competitors (N) raises firms' patenting efforts as complexity of technologies grows.

This result arises because $\frac{\partial \tilde{F}}{\partial N_O}\frac{\partial N_O}{\partial N} > 0$, $\frac{\partial p_i}{\partial N_O}\frac{\partial N_O}{\partial N} < 0$ and $\frac{\partial \epsilon_{\tilde{F}\tilde{f}_i}}{\partial N_O}\frac{\partial N_O}{\partial N} > 0$ as we show in Appendices A.1 and A.2. Then, we can show that:

$$\frac{\partial^2 \tilde{\pi}_i}{\partial o_i \partial N} = \frac{\tilde{f}_i}{\tilde{F}}\Big[\frac{p_i}{\tilde{F}}\Big(\frac{\partial V}{\partial \tilde{F}}\tilde{N}_O - V + \frac{\partial L}{\partial s_i}\Big)\frac{\partial \tilde{F}}{\partial N_O} + \Big[V - \frac{\partial L}{\partial s_i} - \tilde{F}C_a\Big]\frac{\partial p_i}{\partial N_O}\Big]\frac{\partial N_O}{\partial N} > 0, \quad (13)$$

$$\begin{aligned}\frac{\partial^2 \tilde{\pi}_i}{\partial \tilde{f}_i \partial N} = {} & \left[\frac{\partial V}{\partial \tilde{F}} + \frac{\partial^2 V}{\partial \tilde{F}^2}\tilde{F}\epsilon_{\tilde{F}\tilde{f}_i} - C_a\right]\frac{\partial \tilde{F}}{\partial N}\frac{\partial N_O}{\partial N} + \left(\frac{\partial V}{\partial \tilde{F}}\tilde{F} - V + \frac{\partial L}{\partial s_i}\right)\frac{\partial \epsilon_{\tilde{F}\tilde{f}_i}}{\partial N_O}\frac{\partial N_O}{\partial N} \\ & + \Big[\frac{\partial \tilde{F}}{\partial N_O}\frac{p_i}{\tilde{F}} - \frac{\partial p_i}{\partial N_O}\Big]\frac{\partial N_O}{\partial N}\frac{\partial^2 L}{\partial {s_i}^2}\frac{\tilde{f}_i}{\tilde{F}}\left(1 - \epsilon_{\tilde{F}\tilde{f}_i}\right) > 0\,. \end{aligned} \quad (14)$$

if Proposition 1 holds. This shows that more competitors investing at the same rate in technological opportunities have the same effect on patenting incentives as increased investment in technological opportunities by a fixed number of investors.

Next, we can show that:

Proposition 2

Greater technological opportunity reduces firms' patenting efforts as complexity of technologies grows.

To determine the effects of an increase in technological opportunity O we investigate the following cross-partial derivatives:

$$\frac{\partial^2 \tilde{\pi}_i}{\partial o_i \partial O} \quad \text{and} \quad \frac{\partial^2 \tilde{\pi}_i}{\partial \tilde{f}_i \partial O} \quad . \quad (15)$$

If the game set out above is smooth supermodular, it follows from equations (8) and (10) that both cross-derivatives here are negative. To see this note that o_j and O only enter this model as a ratio: an increase in O is equivalent to a reduction in o_j.[7] Equations (8) and (10) are both positive if the game is smooth supermodular. Their signs are determined by the derivatives $\frac{\partial \tilde{F}}{\partial o_j} > 0$ and $\frac{\partial p_i}{\partial o_j} < 0$. The derivatives $\frac{\partial \tilde{F}}{\partial O} < 0$ and $\frac{\partial p_i}{\partial O} > 0$ have exactly opposite signs, reversing the signs of the cross-partial derivatives above.

Therefore, greater technological opportunity lowers firms' overall investments in patenting. It reduces the intensity of competition to dominate individual technological opportunities which lowers investments in facets and the number of new technologies which firms invest in.

Now turn to the effects of an increase in the complexity of technology on firms' incentives to patent. We find that the effect is ambiguous. On one hand F enters our model through the ratio ϕ_i. Therefore, an increase in F is the same as a reduction in f_j, indicating that greater complexity should reduce patenting efforts if Proposition 1 holds. The argument is analogous

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[7] Compare the discussion of the expected number of rivals investing in the same technological opportunity (N_O) in Appendix A.1.

to that made in the case of Proposition 2. On the other hand $\tilde{F}$ also increases directly in F. This second effect counteracts the first as we show below. The following cross-partial derivatives show that it is unclear which effect dominates:

$$\frac{\partial^2 \tilde{\pi}_i}{\partial o_i \partial F} = \frac{\tilde{f}_i}{\tilde{F}} \left(\frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_i} \right) \left[\frac{p_i}{\tilde{F}} \frac{\partial \tilde{F}}{\partial F} - \epsilon_{\tilde{F}\tilde{f}_i} \frac{\partial p_i}{\partial F} \right] \tag{16}$$

$$\frac{\partial^2 \tilde{\pi}_i}{\partial \tilde{f}_i \partial F} = \left[\left(\frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_i} \right) \tilde{F}^{(-1)} (1 - 2\epsilon_{\tilde{F}\tilde{f}_i}) + \frac{\partial^2 V}{\partial \tilde{F}^2} \tilde{F} \epsilon_{\tilde{F}\tilde{f}_i} \right] \frac{\partial \tilde{F}}{\partial F} + \left[\frac{\partial \tilde{F}}{\partial F} \frac{p_i}{\tilde{F}} - \frac{\partial p_i}{\partial F} \right] \frac{\partial^2 L}{\partial {s_i}^2} \frac{\tilde{f}_i}{\tilde{F}} (1 - \epsilon_{\tilde{F}\tilde{f}_i}) \tag{17}$$

Here the terms in round brackets in Equations (16) and (17) are positive if the game is smooth supermodular. These positive terms will determine the sign of both conditions if $\epsilon_{\tilde{F}\tilde{f}_i}$ is sufficiently small. Since we cannot measure this empirically we do not pursue the precise conditions under which complexity raises patenting efforts in this model.

Finally, consider again the case of a technology area in which technology is discrete. Here $F = \tilde{f}_i = 1$ by definition. Additionally, legal costs of defending and exploiting a patent right are no longer a function of the share of patents owned on a technological opportunity, as this share is one by definition. Similarly V no longer depends on the level of applications made: one application guarantees that a firm receives V. Then, firms' payoffs are defined as:

$$\tilde{\pi}_i = o_i V p_i - o_i L - o_i C_o - o_i p_i C_a - C_c(o_i) \quad . \tag{18}$$

A game with this payoff function is no longer supermodular in the sense of Proposition 1. However, we can show that under the slightly stronger assumption that costs of coordinating technological opportunities ($C_c(o_i)$) are strictly convex in the number of opportunities firms invest in, we obtain a unique equilibrium for the game. We can then demonstrate that:

Proposition 3

Greater technological opportunity increases firms' patenting efforts in a discrete technology.

To see that this is true consider the first and second order derivatives of the payoff function with respect to technological opportunities invested in:

$$\frac{\partial \tilde{\pi}}{\partial o_i} = (V - L - C_a)p - \frac{\partial C_c}{\partial o_i} = 0 \qquad \frac{\partial^2 \tilde{\pi}}{\partial {o_i}^2} = -\frac{\partial^2 C_c}{\partial {o_i}^2} \quad . \tag{19}$$

If we assume that costs of coordinating technological opportunities are strictly convex: $\frac{\partial^2 C_c}{\partial {o_i}^2} > 0$, then Proposition 3 can be proved with the help of the implicit function theorem:

$$\frac{\partial o_i}{\partial O} = -\frac{\partial^2 \tilde{\pi}}{\partial o_i \partial O} \bigg/ \frac{\partial^2 \tilde{\pi}}{\partial {o_i}^2} > 0 \quad , \tag{20}$$

where $\frac{\partial^2 \tilde{\pi}}{\partial o_i \partial O} = (V - L - C_a) \frac{\partial p}{\partial O} > 0$. Finally note that this result also implies that:

Corollary 2

An increase in the number of competitors N reduces firms' patenting efforts in a discrete technology.

To see this is true note that $\frac{\partial^2 \tilde{\pi}}{\partial o_i \partial N} = (V - L - C_a) \frac{\partial p_i}{\partial N_O} \frac{\partial N_O}{\partial N} < 0$. Then:

$$\frac{\partial o_i}{\partial N} = - \frac{\partial^2 \tilde{\pi}}{\partial o_i \partial N} \Bigg/ \frac{\partial^2 \tilde{\pi}}{\partial {o_i}^2} < 0 \quad . \tag{21}$$

To conclude our analysis of the model we discuss the relationship of Propositions 2 and 3. The reversal of Proposition 3 as we move from discrete (Equation (18)) to complex technologies (Equation (4)) results from competition over facets in a complex technology. In a discrete technology more rivals or less technological opportunity raise the costs of obtaining granted patents. This reduces patenting efforts as firms cannot make more than one patent application towards a discrete technology. In a complex technology more rivals lead to increased value of technology, raising investments. A reduction in technological opportunity increases patenting efforts as firms seek to maintain their share of granted patents on a given technological opportunity in the face of increased competition by rivals. These mechanisms lead to countervailing patenting incentives in complex and discrete technologies.

3 Dataset and Variables

The model developed in the previous section suggests that technological opportunity and complexity of technology jointly affect firms' patenting behavior. In order to test the predictions of the model developed above we derive measures of technological opportunities and complexity from European patent data. We exploit information on blocking patents provided in these data to derive a new continuous measure of complexity of technologies. This information is also used to construct a measure of fragmentation.[8]

Our empirical analysis is based on the PATSTAT database ("EPO Worldwide Patent Statistical Database") provided by the EPO.[9] We extract all patent applications filed at the EPO between 1980 and 2003: more than 1,5 million patent applications with about 4.5 million referenced documents. Patents are classified using the IPC classification which allows us to analyze sectoral differences in patenting activities. The categorization used is based on an updated version of the OST-INPI/FhG-ISI technology nomenclature.[10] This classification divides the domain of patentable technologies into 30 distinct technology areas.[11] We also classify selected technology areas as discrete or complex using to the classification of Cohen et al. (2000).

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[8] The effects of fragmentation do not emerge directly from our model. We discus the rationale of controlling for this variable below.

[9] We use the September 2006 version of PATSTAT.

[10] See OECD (1994), p. 77

[11] These are listed in Table 8 in the appendix

In the following we discuss our measures of patenting, technological opportunities and complexity. These are the most important variables needed to test the theoretical model. Additionally, we discuss several variables that will be used as control variables in the empirical model of section 5.

Measures of Patenting, Complexity and Technological Opportunity

Number of Patent Applications We compute the number of patent applications A_{iat} filed by applicant i separately for all OST-INPI/FhG-ISI 30 technology areas a on an annual (t) basis. To aggregate patent applications to the firm level two challenges must be overcome: firm names provided in PATSTAT are occasionally misspelled and subsidiaries of larger firms are not identified in the dataset. Therefore, we devoted a considerable amount of resources to clean applicant names and to consolidate ownership structures.[12] The aggregation of patent applications are based on these consolidated applicants' identities. The variables discussed below are also based on this consolidation.

Due to the skew distribution of patent applications we transform the variable logarithmically to derive a dependent variable for estimation. Table 3 shows the transformed variable is much closer to a normally distributed variable than the raw measure of patent applications.

Technological Opportunity In our model, we establish a clear relationship between firms' patenting levels in complex technologies and the emergence of new technological opportunities. Unfortunately, a direct measure of existence or emergence of new technological opportunities does not exist. Instead, we use a construct that is based on the strength of the link between R&D firms conduct within a technology area and relevant basic research as an indirect measure of the emergence of new technological opportunities. This construct is based on the assumption that basic research is more likely to open up new technological opportunities than applied research which predominantly refines existing technologies.

Early stages of the evolution of a technology are characterized by a large share of basic research often conducted in publicly-funded labs. In later stages of a technology, industry driven development of existing technological opportunities will dominate basic research. Then the focus is on refining existing opportunities rather than creating new ones. While there is no perfect measure for the position of a technology area in the stylized cycle of technology evolution, the share of references listed on a patent which point to non-patent literature (mostly scientific publications) can be used as a good proxy for the strength of the science link of a technology (Meyer, 2000, Narin and Noma, 1985, Narin et al., 1997).

Therefore, we use the average number of non-patent references per patent in a technology area as a proxy for the position of a technology area in the technology cycle and hence as a

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[12] The aggregation of patenting activities on the firm levels involved the consolidation of subsidiaries of large corporations. Detailed information on the cleaning and aggregation algorithms can be obtained from the authors upon request. We would like to thank Bronwyn Hall for providing us with software for this purpose. We used this and undertook additional efforts to consolidate firm names.

measure for the creation of new technological opportunities.

Complexity of Technology Areas The distinction between discrete and complex technologies is widely accepted in the literature (Cohen et al., 2000, Kusonaki et al., 1998, Hall, 2005). Discrete technologies are characterized by a relatively strong product-patent link, e.g. in pharmaceuticals or chemistry, whereas in complex industries products are likely to build upon technologies protected by a large number of patents held by various parties. It is often held that patent filing strategies differ strongly between discrete and complex industries. In von Graevenitz et al. (2007) this is shown to be the case for European patent applicants while Hall (2005) demonstrates that the "patenting explosion" in the United States is largely confined to complex technologies.

Despite the widely used notion of technological complexity there is no direct measure of it nor is there an indirect construct related to complexity. Kusonaki et al. (1998) and Cohen et al. (2000) (footnote 44) provide schemes which classify industries as discrete or complex based on ISIC codes. These classification schemes are based on qualitative evidence gathered by the authors from various sources in order to separate different industrial sectors into complex or discrete areas. A major drawback of a classification based on prior information from industry codes is that is does not allow to analyze the influence of different levels of complexity but only to distinguish between discrete and complex industries.

To improve on this, we measure complexity of a technology area through firms' patenting activities. Our measure is derived from the degree of overlap between firms' patent portfolios. Such overlap leads to blocking dependencies among firms. If patents containing prior art critical to the patentability of new inventions in a field are held by two firms, each firm can block its rival's use of new patents. Then, a firm can only commercialize a technology if it receives a license to use such blocking patents. In technology areas in which products draw on many patents -complex technologies- we expect to observe a larger number of such dependencies. In discrete technologies the inverse should be true.

We capture blocking dependencies among firms by analyzing the references contained in patent documents. References to older patents or to non-patent literature are included in EPO patents in order to document the extent to which inventions satisfy the criteria of patentability (Harhoff et al., 2006). Often, existing prior art limits patentability of an invention. For example, the existence of an older but similar invention can reduce the patentability of a newer invention. In these cases *critical* documents containing conflicting prior art are referenced in patent documents and are classified as X or Y references by the patent examiner at the EPO during the examination of the patent application.[13] If the patentability of a firm A's inventions is frequently limited by existing patents of another firm B, it is reasonable to assume that the R&D of A is blocked by B to a certain degree. If the inverse is also true, A and B are in a mu-

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[13] A patent contains various different types of references – not all of them are critical. Often, related inventions which are not critical for the patentability of the invention seeking patent protection are also included in the patent document. The EPO provides a full classification of the references included in patent documents allowing us to identify critical references which are classified as X or Y.

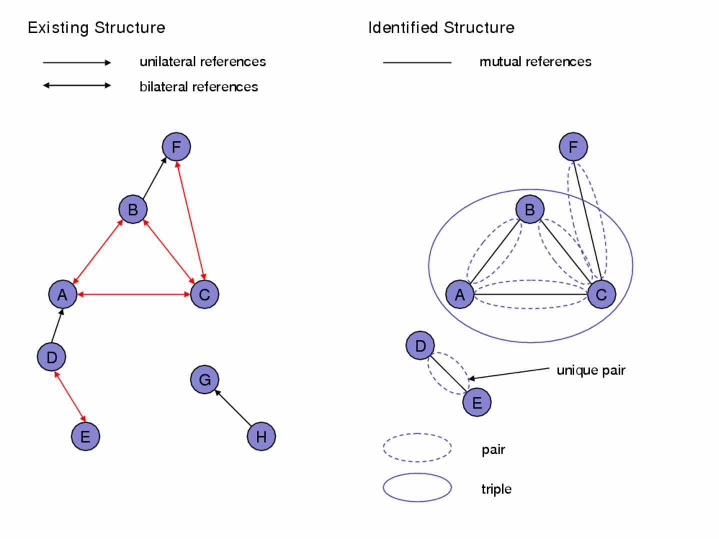

Figure 2: Identification of our measures of a technology field's complexity.

tual blocking relationship which we call a blocking pair. If more than two firms own mutually blocking patents the complexity of blocking relationships increases and resolution of blocking becomes increasingly costly. To capture more complex structures of blocking we compute the number of *Triples* in which three firms mutually block each other's patents. Figure 2 provides a graphical example of our complexity measure.

From a computational perspective, pairs and triples are identified using the following approach: For each firm i we analyze all critical patent references contained in firm i's patents applied for in a technology area a over the current and the two preceding years ($t-2$ to t) and identify the owners of the referenced patent documents. In the next step we keep the most frequently referenced firms (top 20) yielding annual lists of firms which are blocking firm i in year t.[14] Pairs are then established if firm A is on firm B's list of most frequently referenced firms and, at the same time, firm B is on firm A's list of most frequently referenced firms. Finally, triples are formed if firm A and firm B, firm A and firm C and firm B and firm C form pairs in the same period. We include the total number of existing triples $_{at}$ in area a and year t in our regression in order to analyze how the complexity of a technology area influences firms' patenting behavior in this technology area.

Fragmentation of Prior Art Ziedonis (2004) shows that semiconductor firms increase their patenting activities in situations where firms' patent portfolios are fragmented. Ziedonis' fragmentation index has predominantly been studied in complex industries (Ziedonis, 2004, Schankerman and Noel, 2006, Siebert and von Graevenitz, 2008) where increasing fragmentation raises firms' patent applications. This is attributed to firms' efforts to reduce potential hold-up by opportunistic patentees owning critical or blocking patent rights – a situation which is associated with the existence of *patent thickets*.

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[14] The threshold of keeping only the 20 most frequently referenced patent owners is an arbitrary choice. Our results are robust to different choices of the threshold level.

We construct an index of fragmentation of patent ownership for each firm based on the fragmentation index proposed by Ziedonis (2004):

$$Frag_{iat} = 1 - \sum_{j=1}^{n} s_{ijt} \tag{22}$$

where s_{ijt} is firm i's share of critical references pointing to patents held by firm j. Small values of this fragmentation index indicate that prior art referenced in a firm's patent portfolio is concentrated among few rival firms and vice versa. Therefore, the index proxies intensity of competition in technology space (N in the theoretical model).

Unlike previous studies of patenting in complex technologies relying on USPTO patent data (Ziedonis, 2004, Schankerman and Noel, 2006, Siebert and von Graevenitz, 2008) we base the computation of the fragmentation index solely on critical references which are classified as limiting the patentability of the invention to be patented (X and Y references). This distinction is not available in the USPTO data. Computing the fragmentation index based on critical references should yield a more precise measure of hold-up potential.

Control Variables

Technological Diversity of R&D Activities A firm's reaction to changing technological or competitive characteristics in a given technology area might be influenced by its opportunities to strengthen its R&D activities in other fields. For example, if a firm is active in two technology areas it might react by a concentration of its activities in one area if competition in the other area is increasing. If a firm is active in only one technology area, it does not possess similar possibilities to react to increases in competitive pressure. In order to control for potential effects of opportunities to shift R&D resources we measure the total number of technology areas ($\text{Areas}_{i,t}$) with at least one patent application filed by firm i in year t.

Size Dummies. While we do not explicitly model the influence of firm size on patenting behavior, it seems reasonable to assume that the cost of obtaining and upholding a patent depends on the size of a firm. In particular, larger firms might face lower legal cost due to economies of scale, increased potential to source in legal services and accumulation of relevant knowledge which in turn might lead to a different patenting behavior than smaller firms. For instance Somaya et al. (2007), find that the size of internal patent departments positively influences firms' patenting propensity.

If the economies-of-scale argument holds, the cost of patenting should not be directly related to size characteristics such as a firm's number of employees, its total revenues or sales. Rather, the cost of patenting can be assumed to be a function of the total amount of patents filed by a firm. Therefore, we include a 'size dummy' variable based on the number of patents filed by a firm in a technology area in a given year in our regressions. We distinguish between small and large patentees. These size categories are based on annual patent applications in a given area a. Firms belonging to the upper half of the distribution of patentees in a given year are coded as large firms.

4 Descriptive Analysis of Patenting in Europe

In this section we provide descriptive aggregate statistics on patenting trends at the EPO. Discrete and complex technology areas are compared with regard to selected patent indicators. We show that descriptive evidence on patenting supports the theoretical model. Also, the measure of complexity is supported by a comparison with existing measures.

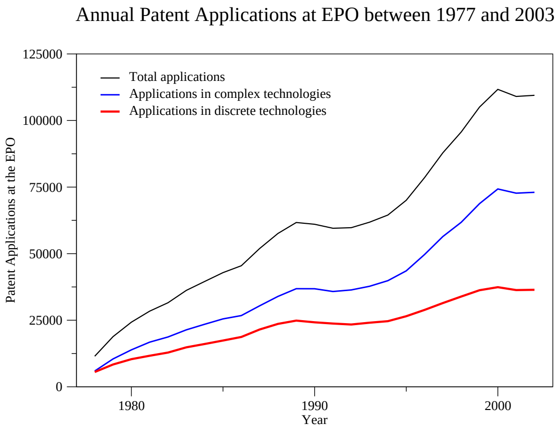

Figure 3: Annual number of patent applications filed at the EPO by priority year. Note: Blue line indicates total patent applications. Red line indicates patent applications in complex technology areas. Green line indicates patent applications in discrete technology areas.

Figure 3 presents annual patent applications filed at the EPO between 1978 and 2003. We distinguish applications filed in complex and discrete technology areas using the categorization of Cohen et al. (2000). The Figure shows patenting grew strongly over this period, with the main contribution coming from technology areas classified as complex. This development is comparable to trends at the USPTO. Hall (2005) shows that the strong increase in patent applications is is driven by firms patenting in the electrical, computing and instruments area all of which are complex technology areas by the classification of Cohen et al. (2000).

Now we turn to explanations for the strong growth in patenting. First, consider a leading explanation for increased patenting in complex technology areas: the fragmentation of patent rights in a complex technology area is likely to raise firms' transactions costs as they must bargain with increasing numbers of rivals in order to prevent hold up of their products. Ziedonis (2004) and Schankerman and Noel (2006) show that increased fragmentation of patents leads to greater patenting efforts in the semiconductor and software industries respectively. Figure 4 provides annual averages of the fragmentation index at the EPO for the years 1980 to 2003.[15] Two observations derived from Figure 4 are striking: First, fragmentation of ownership rights

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[15] The precise definition of this measure is given in Section 3 above.

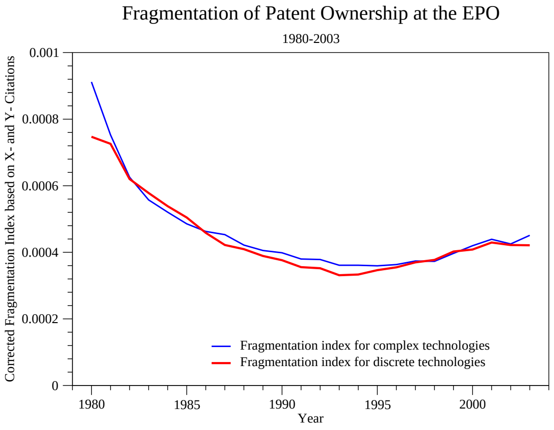

Figure 4: Average fragmentation index. Note: Blue line indicates average level of fragmentation index in complex technology areas. Red line indicates average level of fragmentation index in discrete technology areas.

fell steadily before 1995 and then increased gradually thereafter. Second, the difference in the fragmentation index in complex and discrete technology areas is negligible.

Both observations raise the question whether the growth in patent applications can be attributed to fragmentation alone. While the development of fragmentation in complex and discrete areas is almost identical we observe striking differences in the growth of patent applications between complex and discrete technology areas.

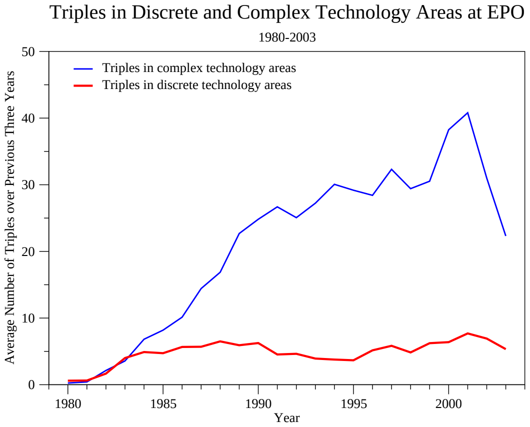

Figure 5: Average number of triples identified. Note: The blue line indicates average number of triples in complex technology areas. The red line indicates average number of triples in discrete technology areas.

Next we explore two explanations for the increase in patenting at the EPO that build on the theoretical model developed above: firstly firms build patent portfolios to strengthen their bargaining positions if complex bargaining situations are more likely to arise and secondly the pressure to obtain patents becomes more intense as technological opportunity declines. The first of these explanations is similar to the explanation for patenting derived from fragmentation of property rights: it also emphasizes transactions costs increases derived from bargaining over blocking patents. However, we believe that transactions costs also rise if a small number of firms own patent rights that depend on the rights of other firms that also block each other. Then, bargaining will become increasingly complex as blocking cannot be resolved through a series of bilateral negotiations. Our measure of mutual blocking between three and more firms (Triples) captures the degree to which complex blocking arises.

In Figure 5 this measure is presented. The Figure presents annual averages of the number of Triples in complex and in discrete areas.[16] We observe very different developments of the count of Triples in these kinds of technology areas. The number of Triples is stable at values well under 10 in discrete technology areas, while it increases strongly in complex technology areas. It is reassuring to see that this measure capturing complex bargaining situations is greater in complex technologies as previously defined by Cohen et al. (2000).

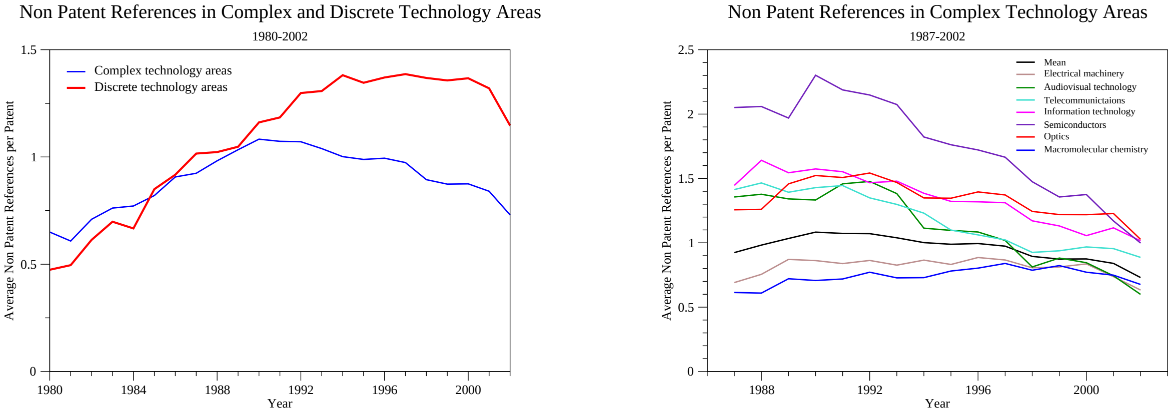

Figure 6: The left panel presents average non patent references per patent for complex (blue line) and discrete (red line) technology areas. The right panel presents average non patent references per patent for several complex technology areas.

Next, consider the development of technological opportunities. Proposition 2 indicates greater technological opportunity in a complex technology should lower the pressure to patent. As noted in Section 3 we measure technological opportunity using changes in the rate of references to non patent literature within a technology area. This measure provides information about variation in technological opportunities between and across technology areas. The left panel of Figure 6 shows a hump shaped pattern for technological opportunities in complex technology industries. In contrast, technological opportunities in discrete technologies also level off, but at a later date then in complex technologies. Note that technological opportunities in complex technology areas began to decline just after 1992, which coincides with the date at which the growth in patent applications at the EPO picked up as Figure 3 shows. The right panel of the Figure shows that average non patent references in complex technology areas mask considerable variation across and especially within technologies.

Table 1 provides additional information on the distribution of triples across all 30 technology areas. It shows the significant hold up potential, measured by Triples, within ICT

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[16] We distinguish complex and discrete using the classification suggested by Cohen et al. (2000) here.

technologies. The number of Triples is between five and six times as large there as in other industries such as Handling, Printing which still exhibit significant complexity by our measure.

Table 1: The Distribution of Triples Between 1987 and 2002

Technology area	**Mean**	**Median**	**Std. dev.**	**Minimum**	**Maximum**
Electrical machinery, Electrical energy	24.23	20	8.99	10	42
Audiovisual technology	116.48	120	17.68	74	148
Telecommunications	99.64	93	39.17	27	166
Information technology	57.16	59	10.71	28	73
Semiconductors	62.84	63	17.89	26	91
Optics	57.30	58	12.02	42	77
Analysis, Measurement, Control	6.61	4	6.31	0	21
Medical technology	4.10	3	2.16	1	8
Nuclear engineering	0.95	1	1.17	0	4
Organic fine chemistry	3.77	2	4.03	0	15
Macromolecular chemistry, Polymers	16.00	14	8.17	4	32
Pharmaceuticals, Cosmetics	3.47	4	2.68	0	8
Biotechnology	0.00	0	0.00	0	0
Agriculture, Food chemistry	0.07	0	0.26	0	1
Chemical and Petrol industry	11.16	10	5.49	4	22
Chemical engineering	1.35	1	0.87	0	3
Surface technology, Coating	3.48	3	2.82	0	9
Materials, Metallurgy	2.41	2	2.12	0	6
Materials processing, Textiles, Paper	3.92	3	2.73	1	9
Handling, Printing	20.26	16	13.55	4	50
Agricultural and Food processing,	0.35	0	0.71	0	2
Environmental technology	3.23	0	4.73	0	15
Machine tools	1.91	1	1.57	0	5
Engines, Pumps and Turbines	21.72	15	21.10	3	69
Thermal processes and apparatus	0.37	0	0.62	0	2
Mechanical elements	2.33	2	2.14	0	7
Transport	16.54	14	12.00	2	50
Space technology, Weapons	0.00	0	0.00	0	0
Consumer goods	0.72	0	1.05	0	4
Civil engineering, Building, Mining	0.00	0	0.00	0	0

5 The Empirical Model and Results

In this section we set out empirical results. To begin with we provide a discussion of our empirical model and discuss descriptives for the sample used. Then we turn to the results from estimation and a discussion of their implications.

5.1 An Empirical Model of Patenting

Building on the results of Section 2 we estimate a reduce form model predicting the level of patent applications filed by a firm in a given year at the EPO. Patent applications are highly persistent as they reflect long term investments in R&D capacity. Therefore, we include a lagged dependent variable in our model. We estimate the following dynamic relationship:[17]

$$A_{i,t} = \beta_0 + \beta_A A_{i,t-1} + \beta_{AC} A_{i,t-1} C_{i,t} + \beta_O O_{i,t} + \beta_C C_{i,t} + \beta_{OC} O_{i,t} C_{i,t} + \boldsymbol{\beta}_X{}' \boldsymbol{X}_{i,t} + \chi_i + \zeta_{it} \ , \tag{23}$$

where:

$A_{i,t}$ – ln(Patent Applications)

$O_{i,t}$ – Technological Opportunity: Non Patent References

$C_{i,t}$ – Complexity: Triples

$\boldsymbol{X}_{i,t}$ – Control variables: Fragmentation, Area count, Size

This specification allows us to simultaneously control for effects of technological opportunity β_O and complexity β_C and to analyze whether the effect of technological opportunity differs in discrete and complex technologies by interacting both variables ($O_{i,t}C_{i,t}$). We allow the effect of the lagged dependent variable to differ in complex and discrete technology areas (β_{AC}). Our theoretical model shows that patenting behaviour in discrete and complex technology areas is driven by different mechanisms. Therefore, we must allow for different effects of the lagged dependent variable in complex and discrete technology areas as the dynamics of patenting are unlikely to be identical in both kinds of technology areas.

In an extension of this basic specification we also include interaction terms that allow us to distinguish the patenting behavior of large and small firms in complex and discrete technologies. Our theoretical model indicates that firms' patenting behavior will depend on the share of patents they expect to receive on a given technological opportunity which may differ systematically between large and small firms.

Estimates of this specification provide a test of the following hypotheses. These reflect Propositions 2- 3 and Corollaries 1- 2:

H1 : Increased competition by rivals raises the level of patent applications as technologies become more complex (Corollary 1);

H2 : Increased technological opportunity lowers the level of patent applications as technologies become more complex (Proposition 2);

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[17] Our model did not explicitly account for dynamic aspects of firms' strategic decisions. However, it seems appropriate to take the persistent nature of patenting decision into account when analyzing patenting over time.

H3 : Increased competition by rivals lowers the level of patent applications in discrete technologies (Corollary 2);

H4 : Increased technological opportunity raises the level of patent applications in discrete technologies (Proposition 3).

Applying these hypotheses to our specification it may be shown that Hypothesis 2 implies that $\beta_{OC} < 0$ and Hypothesis 4 implies that $\beta_O > 0$. We seek to proxy the effects of greater competition through the use of a Fragmentation index included in the control variables of the above specification. Defining coefficients for an interaction of the complexity measure with Fragmentation as β_{FC} and for Fragmentation alone as β_F, our Hypotheses imply that: $\beta_{FC} > 0$ (Hypothesis 1) and $\beta_F < 0$ (Hypothesis 3).

5.2 Descriptive Statistics for the Sample

Our dataset contains observations of patent applications by firms in specific technology areas and covers the period between 1978 when the EPO began operating and 2003. We intend to study patent applicants patenting over a prolonged period and possibly across several technology areas. Therefore, we excluded small patentees from the sample. Two criteria were used: first, we excluded all those patentees with fewer than 100 patent applications between 1980 and 2003. Second, we excluded those patentees who had fewer than three years of positive patent applications in a technology area in the fifteen years after 1987.

Table 2: Panel Descriptives for the Sample

Firm level (n=2074)	**Mean**	**Median**	**SD**
Total patents	628.27	205	1944.94
Total patents (annual)	37.02	12	111.65
Technological areas (annual)	5.54	4	4.56
Area-Year level (n=650)	**Mean**	**Median**	**SD**
Total patents in area	2594.23	2310	1778.87
Total patents in sample	1449.35	1012	1695.86
Total firms in area	1077.62	893	668.14
Total firms in sample	266.84	263	253.71
Triples	14.67	2	27.69
Non Patent References	0.98	0.75	0.75
Fragmentation	0.001	0	0.009

These criteria result in a sample containing 182,524 observations of patent applications by a firm in a technology area. Table 2 shows that these patent applications are due to 2074 distinct firms. The average size of these firms' patent portfolios in 2003 was 628 patents resulting from an average of 37 patent applications per firm and year across all technology areas. 34% of observations in the dataset contain a zero patent application count but only 0.05% of observations belong to firms that have no patent applications at all in a given year.

We treat firms operating in several technology areas as distinct in each area. Hence, our panel structure is not defined over firms' total patent applications per year (firm-years) but over firms' annual patent applications within specific technology areas (firm-area-years). We do this to control for area specific patenting behavior of individual firms and its relation to area characteristics like complexity. Where we use panel data, the panel is unbalanced due to entry and exit of firms into technology areas. The lower half of Table 2 shows that our sample covers on average 55.8% of the annual mean of 2594 patent applications filed within an average technology area. As our sample focusses on large patentees it is not surprising that the share of firms we cover in our analysis is smaller: we observe 24.8% of patentees at the EPO between 1978 and 2003 (see Table 2).

Table 3: Descriptive Statistics for the Sample (1987-2002)

Variable	**Aggregation level**	**Mean**	**Median**	**Standard deviation**	**Minimum**	**Maximum**
Patent applications	Firm	5.431	1.000	18.594	0.000	752.000
$\log$ Patent applications	Firm	1.051	0.693	1.052	0.000	6.624
Areas	Firm	8.751	7.000	6.027	0.000	30.000
Large dummy	Firm	0.504	1.000	-	0.000	1.000
Non Patent References	Area	1.151	0.894	0.827	0.174	4.532
Triples	Area	18.480	5.000	30.085	0.000	166.000
Fragmentation	Area	0.001	0.000	0.006	0.000	0.355

Observations = 173,448

Sample statistics for 1992

Patent applications	Firm	4.235	1.000	14.024	0.000	387.000
$\log$ Patent applications	Firm	0.923	0.693	0.990	0.000	5.961
Areas	Firm	7.746	6.000	5.563	0.000	27.000
Large dummy	Firm	0.438	0.000	-	0.000	1.000
Non Patent References	Area	1.205	0.970	0.747	0.290	3.554
Triples	Area	15.761	3.000	25.348	0.000	104.000
Fragmentation	Area	0.001	0.000	0.006	0.000	0.168

Observations = 11,325

Table 3 presents descriptive statistics at the firm-area-year level. Most firms in the sample patent relative broadly across technology areas. While the number of patent applications within a given technology area is relatively low with 5.43 application per year firms are active in 8 or 9 different technology areas. The average technology area contained about 18.5 Triples in a given year – however the distribution is skew with a median of 5 and a maximum of 166 Triples (observed in Telecommunications in 2000). The level of non patent references in the average technology area is 1.151. Table 3 also contains information about sample statistics for the year 1992, after which patent applications increased markedly as Figure 3 shows. A comparison of sample means (upper part of Table 3) and means for 1992 (lower part of 3) shows that firms patent in more areas, face more complexity (Triples) and generate fewer non patent references after 1992 than before. This confirms what we showed previously.

5.3 Results

In this section we present the results from estimation of the empirical model (Equation 23). This section focuses on results from GMM estimation of the model. Using GMM we instrument the lagged dependent variable as well as several of our explanatory variables which may be expected to be endogenous. We provide two tables with models of varying complexity. These show that the predictions of the theoretical model in Section 2 are supported by the data. Finally, we provide a table which shows how strong the effects of complexity and technological opportunity are in different technology areas.

Table 4 presents results of system GMM estimators using forward deviations transformations (Blundell and Bond, 1998, Arellano and Bover, 1995, Alvarez and Arellano, 2003).[18] Reported standard errors are based on two step estimators using the correction suggested by Windmeijer (2005). Tests for first, second and third order serial correlation (m1-m3) indicate presence of first and second order serial correlation. In all specifications we instrument predetermined variables with third order lags and endogenous variables with fourth order lags. Instrument sets are collapsed in order to reduce the number of instruments used.

Specification SGMM A contains the lagged dependent variable, measures of technological opportunity (*NPR*), complexity (*Triples*), the breadth of a firms' activities within the patent system (*Areas*), a dummy for the size of a firms' patent portfolio (*Large*) as well as dummies for year and main technology area. Additionally, SGMM B contains a corrected measure of fragmentation. Hansen tests for both of these simple specifications reject their validity.

Specification SGMM C includes interactions of the complexity measure (*Triples*) with the lagged dependent variable and the measure of technological opportunity (*NPR*). This specification performs much better, the χ^2 statistic being lower than for the previous specifications.[19]

In specification SGMM FULL *Fragmentation* is interacted Triples, to capture the hypothesis that more competition affects patenting differently in complex and discrete technologies. The specification represents an improvement over the previous in terms of the Hansen test. Finally, specification SGMM L includes interactions which test the effects of firm size on non patent references. This specification performs best, the Hansen test does not reject the model.

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[18] All models were estimated with `xtabond2` in Stata 9.2 . This package is described in Roodman (2006).

[19] In unreported results we find the model improves through the combination of both interaction effects reported. This indicates that the interactions capture an important aspect of the data generating process.

Table 4: Patent Applications Estimates

	(1)	(2)	(3)	(4)	(5)
Variable	SGMM A	SGMM B	SGMM C	SGMM FULL	SGMM L
log Patentcount$_{t-1}$	0.777***	0.709***	0.485***	0.533***	0.678***
	(0.042)	(0.047)	(0.074)	(0.087)	(0.068)
log Patentcount$_{t-1}$ × Triples			-0.015***	-0.016***	-0.015***
			(0.002)	(0.002)	(0.002)
Non Patent References (NPR)	0.216***	0.191***	1.525***	1.613***	1.386***
	(0.031)	(0.032)	(0.190)	(0.241)	(0.182)
NPR × Triples			-0.041***	-0.043***	-0.034***
			(0.004)	(0.005)	(0.004)
NPR × Triples × Large					0.006***
					(0.001)
NPR × Large					-0.425***
					(0.052)
Fragmentation		5.685*	-4.606	-13.208	-12.482*
		(2.309)	(4.608)	(9.279)	(6.192)
Fragmentation × Triples				0.305**	0.247*
				(0.114)	(0.097)
Triples	-0.000	-0.000	0.069***	0.072***	0.057***
	(0.000)	(0.000)	(0.007)	(0.008)	(0.006)
Areas	0.059***	0.066***	0.115***	0.113***	0.096***
	(0.007)	(0.007)	(0.012)	(0.013)	(0.010)
Large	-0.115***	-0.094***	0.042	0.031	0.409***
	(0.027)	(0.027)	(0.054)	(0.061)	(0.081)
Year dummies	YES	YES	YES	YES	YES
Primary area dummies	YES	YES	YES	YES	YES
Constant	-0.358***	-0.357***	-1.531***	-1.625***	-1.515***
	(0.041)	(0.041)	(0.177)	(0.223)	(0.167)
N	173448	173448	173448	173448	173448
m1	-25.48534	-21.6864	-10.69893	-9.690637	-13.49454
m2	18.08254	15.15458	2.488548	2.477419	5.564835
m3	-1.650511	-1.709003	1.143266	1.446003	.7390595
Hansen	566.1257	558.1005	29.0312	20.61644	10.67657
p-values	0.00000	0.00000	0.00000	.00095	.05818
Degrees of freedom	4	4	5	5	5

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

1. Asymptotic standard errors, asymptotically robust to heteroskedasticity are reported in parentheses
2. m1-m3 are tests for first- to third-order serial correlation in the first differenced residuals.
3. Hansen is a test of overidentifying restrictions. It is distributed as χ^2 under the null of instrument validity, with degrees of freedom reported below.

4. In all cases GMM instrument sets were collapsed and lags were limited.

We find that greater technological opportunities raise patenting levels. This effect is highly significant across all estimated specifications (see Columns (1) to (5) of Table 4). The inclusion of the interaction between or measure of complexity (Triples) and technological opportunities shows that the effect differs in discrete - and complex technologies. In particular, if the number of triples in a technology area is larger than 37 (in specifications (3) and (4)) or larger than 40 in specification (5) of Table 4, the overall effect from increasing technological opportunities is negative as $\beta_O + \beta_{OC} \times C_{i,t} < 0$. The negative coefficient on the interaction of complexity and non patent references supports Hypothesis 2: increasing technological opportunities reduce patenting efforts in more complex technology areas.

Returning to the overall effect of increased technological opportunities Table 1 shows the average number of Triples for 5 technology areas in our sample is greater than 40. For Audiovisual technology and Optics triples are always above 40. This indicates that increased technological opportunities always or almost always reduce patenting efforts in these areas. In case of larger firms the predicted effects of complexity arise when the number of Triples is above 4. This is always the case for 9 technology areas in our sample. The effect is further strengthened for large firms as $\beta_{OCL} \times C_{i,t} \times L_{i,t} + \beta_{OL} \times L_{i,t} < 0$ in Column (5) of Table 4. Since $\beta_O + \beta_{OC} \times C_{i,t} > 0$ for areas with fewer Triples (even in the case of large firms) Hypothesis 3 can not be rejected.

With regard to the effects of the number of competitors blocking a specific firm in technology space we confirm both Hypotheses 1 and 3. We find that increased Fragmentation reduces firms' patenting efforts in discrete technologies, while it raises them increasingly the more complex a technology area becomes. It is interesting to note that it takes between 43 and 50 Triples for the effects of Fragmentation to affect patenting positively in specifications (4) and (5). These levels are similar to those resulting from interactions of complexity and technological opportunity and indicate that patenting behaviour changes significantly once a technology area contains more than 40 triples.

In a next step, we test the robustness of our results using alternative GMM estimators. Results from these tests are reported in Table 5. Here, we vary size of the instrument set and the estimator used. All models reported in Table 5 are estimated using forward deviations and reported standard errors corrected as previously noted. The models differ in the number of overidentifying restrictions employed as well as assumptions about the correlation of the explanatory variables with fixed effects. The four models reported in the central part of the table allow for correlation between all explanatory variables apart from *Triples* with fixed effects. In the two specifications on the right side of the table we assume that subsets of the explanatory variables are uncorrelated with fixed effects.

The number of observations in our dataset implies that $T/N \rightarrow 0$. Therefore, a systems GMM estimator (Blundell and Bond, 1998) using forward deviations is asymptotically consistent (Alvarez and Arellano, 2003, Hayakawa, 2006). We employ this estimator as the patenting series are highly persistent in our sample: the coefficient on the lagged dependent variable in an AR1 model with time and primary area dummies is 0.92. Blundell and Bond (1998) note that difference GMM is affected by a weak instruments problem in this context.

Table 5: Robustness Checks for Patent Applications Estimates

	Allowing correlation with fixed effects				Assuming no correlation with fixed effects	
Variable	SGMM MIN	SGMM L	DGMM L	SGMM L2	SGMM NPR	SGMM F
log Patentcount$_{t-1}$	0.684***	0.678***	0.863***	0.735***	0.715***	0.915***
	(0.072)	(0.068)	(0.091)	(0.058)	(0.047)	(0.039)
log Patentcount$_{t-1}$× Triples	-0.017***	-0.015***	-0.012***	-0.011***	-0.007***	-0.004***
	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)
Non Patent References (NPR)	1.581***	1.386***	1.198***	0.968***	0.271***	0.171
	(0.221)	(0.182)	(0.164)	(0.113)	(0.019)	(0.119)
NPR × Triples	-0.038***	-0.034***	-0.028***	-0.024***	-0.008***	-0.003
	(0.005)	(0.004)	(0.004)	(0.002)	(0.001)	(0.003)
NPR × Triples × Large	0.006***	0.006***	0.006***	0.005***	0.004***	0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
NPR × Large	-0.436***	-0.425***	-0.262***	-0.397***	-0.466***	-0.506***
	(0.055)	(0.052)	(0.033)	(0.042)	(0.034)	(0.032)
Fragmentation	-15.234*	-12.482*	-13.998*	-4.848	-1.448	-2.313
	(6.510)	(6.192)	(6.123)	(3.654)	(1.210)	(1.946)
Fragmentation × Triples	0.262**	0.247*	0.181*	0.188*	0.102*	0.156*
	(0.100)	(0.097)	(0.091)	(0.083)	(0.044)	(0.071)
Triples	0.063***	0.057***	0.042***	0.040***	0.015***	0.007
	(0.007)	(0.006)	(0.005)	(0.004)	(0.001)	(0.004)
Areas	0.095***	0.096***	0.031*	0.086***	0.085***	0.058***
	(0.010)	(0.010)	(0.014)	(0.008)	(0.007)	(0.006)
Large	0.430***	0.409***	0.257***	0.325***	0.442***	0.412***
	(0.087)	(0.081)	(0.053)	(0.065)	(0.049)	(0.048)
Year dummies	YES	YES	YES	YES	YES	YES
Primary area dummies	YES	YES	YES	YES	YES	YES
Constant	-1.672***	-1.515***		-1.151***	-0.597***	-0.526***
	(0.198)	(0.167)		(0.105)	(0.046)	(0.106)
N	173448	173448	171380	173448	173448	173448
m1	-12.75267	-13.49454	-9.115675	-16.66536	-20.32686	-28.27661
m2	4.690134	5.564835	5.686894	9.293913	12.525	20.07668
m3	1.093296	.7390595	-.4191068	-.4131314	-1.354271	-1.478497
Hansen	2.178791	10.67657	7.067067	70.62775	184.0212	288.5185
p-values	0.1399	0.0582	0.1324	0.0000	0.0000	0.0000
Degrees of freedom	1	5	4	9	7	7

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

1. Asymptotic standard errors, asymptotically robust to heteroskedasticity are reported in parentheses
2. m1-m3 are tests for first- to third-order serial correlation in the first differenced residuals.
3. Hansen is a test of overidentifying restrictions. It is distributed as χ^2 under the null of instrument validity, with degrees of freedom reported below.
4. In all cases GMM instrument sets were collapsed and lags were limited.

Specification DGMM L reported in Table 5, estimated by difference GMM, does not suggest this problem is severe here. The coefficient on the lagged dependent variable is somewhat above that reported for the comparable systems estimators. It is also significantly above the coefficients from the OLS regressions reported in Table 7. Therefore, we focus our analysis on the results from the system estimators. The substantive results provided by the difference estimator are the same as those from the systems estimators.

In all models reported in Table 5 the instrument sets were collapsed[20] and instrumenting lags were limited as described below. This was done as the Hansen test and difference in Hansen tests rejected the overall instrument sets as well as individual instruments where larger instrument sets were employed. Specification SGMM L2 illustrates how sensitive the Hansen test is to the size of the instrument set here. This specification is identical to SGMM L, we just allow for an extra lag on the instrument sets for the endogenous variables in this specification. The specification is rejected by the Hansen test.

All models reported in Table 5 contain the following explanatory variables: *Non patent references*, *Triples*, *Fragmentation*, *Area count*, *Large dummy* and the lagged dependent variable as well as interactions of some of these variables. We consider *Large* and *Area count* to be endogenous as they reflect decisions about how widely and where to engage in research which may be contemporaneous with decisions determining the level of patent applications. We consider the remaining variables to be predetermined since they depend in large part on the aggregated decisions of rival firms. Finally note that we include only year and primary area dummies as well as *Triples* in the levels equation as it is likely that the fixed effects are correlated with differences in the remaining explanatory variables. *Triples* is the only variable that reflects purely technology area specific characteristics which may be assumed to be orthogonal to firm specific effects.

We estimate two models in which we treat Fragmentation (GMM F) and Non patent references (GMM NPR) as uncorrelated with fixed effects. Results from the Hansen tests for both specifications reported in Table 5 show that these models are clearly rejected.

Our preferred models are reported as SGMM MIN and SGMM L in Table 5. In SGMM MIN we restrict the number of instruments such that the model is just overidentified. Hayakawa (2006) argues that such a minimum instruments specification is unbiased in settings where T is fixed and $N \to \infty$. Specification SGMM L includes one additional lag for the endogenous variables. Results from these two specifications are statistically indistinguishable.

Based on this specification Table 6 provides effects of changes in complexity (*Triples*), technological opportunities (*Non patent references*) and *Fragmentation* for patenting rates in

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[20] Collapsing instrument sets reduces the number of moment conditions used for GMM (Roodman (2006)).

nine technology areas.[21] The table presents effects for small and large firms where appropriate. Five of the technology areas presented are highly likely complex as the mean level of Triples is clearly above 42 in these areas (viz. Table 1). They are Audiovisual Technology, Telecommunications, Information Technology, Semiconductors and Optics. We also present results for five additional areas. These are more likely discrete by this measure: Medical Technology; Electrical Machinery; Analysis, Measurement, Control; and Pharmaceuticals. Our theoretical predictions are borne out by specification SGMM L and Table 6. First, we find that in discrete technologies additional technological opportunity raises firms' patenting rates. The coefficient for *Non patent references* is positive and highly significant. Even in case of large firms the overall effect remains positive. This supports our finding that Hypothesis 4 cannot be rejected.

Table 6: Mean and Median Percentage Changes in Patent Applications in Complex and Discrete Technologies

Technology area	**Applications growth 1990-2000**	**Triples SD change**		**Non patent references SD change**		**Fragmentation**	
		Small	**Large**	**Small**	**Large**	**Unit change (+0.0001)**	**SD change**
Complex Technologies							
Telecommunications	253%	-3,74%	24,52%	-34,37%	-31,81%	0,13%	10,21%
		2,82%	44,03%	-29,92%	-28,03%	0,10%	8,24%
Information Technology	174%	-2,88%	4,45%	-10,60%	-11,84%	0,02%	1,48%
		1,65%	9,32%	-10,94%	-12,12%	0,02%	1,59%
Semiconductors	63%	-24,82%	-9,09%	-24,42%	-25,86%	0,03%	2,44%
		-21,73%	-4,90%	-25,01%	-26,34%	0,03%	2,57%
Audiovisual Technology	52%	6,64%	18,66%	-50,69%	-46,79%	0,17%	21,66%
		16,42%	29,61%	-51,72%	-47,71%	0,17%	22,48%
Optics	41%	-4,79%	4,85%	-7,69%	-8,70%	0,02%	1,20%
		0,90%	11,12%	-7,84%	-8,83%	0,02%	1,26%
Discrete Technologies							
Pharmaceuticals	221%	-13,59%	-9,80%	49,02%	30,35%	-0,12%	-4,53%
		-13,99%	-10,16%	48,97%	29,92%	-0,11%	-4,50%
Medical Technology	148%	6,85%	7,32%	5,69%	3,45%	-0,11%	-5,13%
		7,55%	8,04%	5,69%	3,48%	-0,11%	-5,16%
Electrical Machinery	91%	12,43%	17,42%	3,51%	1,72%	-0,06%	-2,46%
		17,35%	22,64%	4,55%	2,56%	-0,08%	-2,89%
Analysis, Measurement, Control	75%	1,94%	6,67%	10,35%	5,82%	-0,11%	-2,34%
		5,02%	9,80%	10,35%	6,62%	-0,12%	-2,54%

This table reports **means** (upper row) and **medians** (lower row) for each technology area. We report changes in patent applications in response to standard deviation (**SD**) changes in each variable. For *Triples* and *Non patent references* we report effects for **small** and **large** firms.

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[21] These effects are calculated taking account of the logarithmic transformation of the dependent and the lagged dependent variable.

Second, the coefficient on the interaction of *Non patent references* and *Triples* is negative. The overall effect of additional Non patent references on patenting becomes negative if there are more than 42 Triples in a technology area. As Table 6 shows the effects of increases in *Non patent references* on the level of patenting are substantial in the technology areas we identify as complex. These findings show that Hypothesis 2 cannot be rejected. Turning to the effects of complexity we find that the coefficient on *Triples* is positive and greater than that on the sum of interactions of *Triples* with *Non patent references* and $Patentcount_{t-1}$. This shows that greater blocking complexity and therefore greater complexity of a technology area increase firms' levels of patenting. Table 6 shows that this result generally holds at the median and at the mean for large firms in complex technology areas apart from Semiconductors.[22] In these areas the mean of $Patentcount_{t-1}$ and *Triples* is often significantly greater than the median, indicating that the mean firm is usually a large firm.

Interestingly, Table 6 also shows that the effect of *Fragmentation* on firms' patenting efforts in complex technology areas is positive confirming Hypothesis 1. Also, *Fragmentation* has a negative effect on patenting in discrete technology areas, further confirming Hypothesis 3. The positive effects for complex technology areas support the findings of Ziedonis (2004), Schankerman and Noel (2006) and Siebert and von Graevenitz (2008) who find that additional fragmentation of patent ownership increases patenting efforts in the semiconductors and software in the United States.

Additionally to the GMM results reported here, Table 7 (Appendix B) provides results from OLS on the pooled sample and from fixed effects regressions. These results are known to be biased due to inclusion of the lagged dependent variable. However, they provide lower and upper bounds on the values of the lagged dependent variable for GMM (Bond (2002)). We find the coefficient of the lagged dependent variable in the models SGMM C and SGMM FULL lies within the range given by results of OLS on a pooled sample and a fixed effects model. In case of SGMM L the coefficient of the lagged dependent variable is marginally greater than the results of OLS estimation.

Finally, our results on the interaction of the lagged dependent variable with *Triples* indicate that persistence of patenting decreases as technology areas become more complex. This suggests patentees are more responsive to their competitors' patenting behavior in complex technology areas than in discrete technology areas.

6 Conclusion

Patent applications have been increasing steeply at the USPTO and the EPO since 1984 and 1992 respectively. In both cases these increases have raised questions about the operations of the affected patent offices as well as effects of these trends on economic activity more generally (F.T.C., 2003, National Research Council, 2004, von Graevenitz et al., 2007,

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[22] The precise delineation of the areas for Information Technology and Semiconductors in the classification we use is not clear. In von Graevenitz et al. (2007) we find that a large proportion of patents from semiconductor firms are patented within the Information Technology area.

Bessen and Meurer, 2008). Our paper makes a number of contributions towards a systematic explanation of these phenomena. There is strong evidence by now that patenting has increased in response to evolution of the legal environment, specifically in the United States, to changes in the management of R&D and patenting, and to increasing complexity of technology and more strategic behavior of patent applicants (Kortum and Lerner, 1998, Hall and Ziedonis, 2001, Ziedonis, 2004). But the contribution of technological opportunity to current patenting trends and its interaction with other determinants has been less well understood.

This latter effect is central to our analysis. Our model is the first to consider the effect of complexity and of technological opportunity *jointly*. Moreover, while other studies have focused on selected industries, our model and the empirical test encompass discrete and complex technologies, providing predictions for patenting behavior in both types of technology. We show theoretically that greater technological opportunity will raise patenting in discrete technologies but will lower it as technologies become increasingly complex. Additionally, we show that greater competition in R&D raises firms' patenting levels in complex technologies.

To test our model we derive a new measure of complexity of blocking relationships in patent thickets. This measure exploits information on critical references to capture mutual blocking between the patent portfolios of firms contained in European patent data. Using the measure we are able to confirm that blocking is a much more serious problem in complex technology areas than in discrete technology areas. We also exploit information on critical references to provide a sharper measure of fragmentation than has been available using data from the USPTO. Finally we make use of references to non patent literature to obtain a metric for the extent of technological opportunities.

Using data on patenting in Europe and these new measures, we find that patenting behavior largely conforms to the predictions of our theoretical model. Most importantly, our results demonstrate that variation in technological opportunity has had important effects on firms' patenting levels in Europe. Our data show that increased technological opportunity during the early 1990's retarded the onset of the patenting explosion that is observable after 1994 when opportunities started to decrease. We also show - for the first time with European data - that greater fragmentation of patent ownership has positive effects on patenting levels in complex technologies (Ziedonis, 2004). Finally we find that greater complexity of technology raises patenting levels.

Using measures of complexity and technological opportunity, we are able to show that patent thickets exist in nine out of thirty technology areas at the EPO. The data indicate that the extent of patent thickets at the EPO has been increasing in recent years. These increases are concentrated in complex technology areas (Hall, 2005, von Graevenitz et al., 2007). Resulting increases in transactions costs would therefore affect exactly those technologies that have been central to large productivity increases in the recent past (Jorgenson and Wessner, 2007). Extended "patent wars" may threaten this source of productivity gains in the long run. In future work we therefore intend to investigate whether strategic patenting has measurable effects on the productivity of firms' R&D investments and how the decision variables of patent offices (fees and administrative rules) might be used to influence patent filings.

While we provide some evidence on the level of complexity of blocking relationships in specific technologies here, open questions remain. In future work we intend to investigate in more detail to what extent technology areas have become more complex over time. Using extensions of the complexity measure introduced here we will seek to characterize these trends in greater detail than was possible here.

Our findings on the effects of technological opportunity raise important questions about the relationship between patent breadth, the fecundity of research areas and firms' R&D investments. We find that the contest for patent rights becomes more intense as the level of technological opportunities decreases if a technology is complex. This raises the question how firms' incentives to patent more intensively interact with incentives to undertake basic research which might stem the reduced fecundity of these technologies. At a more fundamental level the findings indicate that research into the relationship between technological opportunities and R&D is important if we are to understand the welfare implications of recent patenting trends better.

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Appendix

A Technical Appendix for the Theoretical Model

In this section we derive several of the results which we make use of in deriving our theoretical predictions in Section 2. In particular we describe the functions describing the expected number of facets covered $\tilde{F}$ and the probability of patenting a facet p_i.

Note that below we also employ the following definitions:

$$\omega_j \equiv o_j / O \qquad \phi_j \equiv f_j / F \quad . \tag{24}$$

A.1 The Expected Number of Rival Investors

Here we derive the expected number of rival firms N_O that undertake R&D on the same technology opportunity as firm i. This expected number of rivals can be expressed as a sum of products. Each product gives the probability that a given number of rivals invest in the same technological opportunity. All of these probabilities are then summed to give the overall expected number of rival firms on a given technological opportunity:

$$\begin{aligned} N_O &= \prod_{j=1}^{N-1}(1-\omega_j) + 2\binom{N-1}{1}\prod_{j=1}^{N-2}(1-\omega_j)\omega_j + 3\binom{N-1}{2}\prod_{j=1}^{N-3}(1-\omega_j)\prod_{j=N-3}^{N-1}\omega_j \ldots \\ &= \sum_{i=0}^{N-1} i\binom{N-1}{i}\prod_{j=0}^{N-1-i}(1-\omega_j)\prod_{k=0}^{i}\omega_k \end{aligned} \tag{25}$$

It can be shown that N_O is increasing in ω_j. First rewrite N_O as a function of ω_k and ω_j.

$$N_O = \sum_{i=0}^{N-1}\left[(1-\omega_k)i + \omega_k(i+1)\right]\binom{N-1}{i}\prod_{j=0}^{N-2-i}(1-\omega_j)\prod_{l=0}^{i}\omega_l \tag{26}$$

Next we take the derivative:

$$\frac{\partial N_O}{\partial \omega_k} = \sum_{i=0}^{N-1}\omega_k\binom{N-2}{i}\prod_{j=0}^{N-2-i}(1-\omega_j)\prod_{l=0}^{i}\omega_l > 0 \; . \tag{27}$$

An increase in the number of opportunities o_j which other firms invest in, increases the expected number of rivals patenting facets on the same technological opportunity.

A.2 The Expected Number of Facets Covered

The expected number of facets covered through the joint efforts of all firms investing in a technological opportunity is:[23]

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[23] We are grateful for the help of Professor Helmut Küchenhoff and Mr. Fabian Scheipl in deriving this expression.

$$\tilde{F} = F\Big[1 - (1-\phi_i)\prod_{j=1}^{N_O}(1-\phi_j)\Big] \tag{28}$$

As noted above, the derivatives of this expression with respect to F and f_k are important for the results there. Both of these can be shown to be positive:

$$\frac{\partial \tilde{F}}{\partial F} = 1 - (1-\phi_k)^{(N_O-1)}\,(1+\phi_k(N_O-1)) \geq 0\ , \quad \frac{\partial \tilde{F}}{\partial f_i} = \prod_{j=1}^{N_O}(1-\phi_j) > 0\ , \tag{29}$$

where we impose symmetry in the choice of f_k across firms in the derivative w.r.t. F. This derivative is used for comparative statics purposes, after first derivatives have been taken. Therefore, this is necessary. Note that the second derivative above implies that $\frac{\partial^2 \tilde{F}}{\partial f_k \partial F} = 0$.

Finally note that the elasticities of $\tilde{F}$ with respect to F and $\tilde{f}_i$ are:

$$\epsilon_{\tilde{F}\tilde{f}_j} = \phi_j \frac{\Big[1 - \prod_{j=1}^{N_O-1}(1-\phi_j)\Big]}{1-\prod_{j=1}^{N_O}(1-\phi_j)} = \frac{\phi_j}{1-\phi_j}\Big[1 - \frac{\phi_j}{1-(1-\phi_j)^{N_O}}\Big] \leq 1 \quad , \tag{30}$$

$$\epsilon_{\tilde{F}F} = \frac{1-(1-\phi_k)^{(N_O-1)}\,(1+\phi_k(N_O-1))}{1-(1-\phi_k)^{N_O-1}} = 1 - \frac{(1-\phi_k)^{(N_O-1)}\,\phi_k(N_O-1)}{1-(1-\phi_k)^{N_O-1}} \quad , \tag{31}$$

which shows that $1 \geq \epsilon_{\tilde{F}F} \geq 0$ as the denominator in the fraction is always greater than the numerator. This follows from the fact that $\frac{\partial \tilde{F}}{\partial F} \geq 0$.

A.3 The Probability of Patenting a Facet

Now turn to the probability of obtaining a patent on a facet given N_O:

$$\begin{aligned} p_i &= \prod_{j=1}^{N_O-1}(1-\phi_j) + \frac{N_O-1}{2}\cdot\phi_j\prod_{j=1}^{N_O-2}(1-\phi_j) + \frac{(N_O-2)(N_O-1)}{6}\prod_{j=1}^{N_O-3}(1-\phi_j)\prod_{j=N_O-3}^{N_O-1}(\phi_j)...\ , \\ &= \sum_{i=0}^{N_O-1}\frac{1}{i+1}\binom{N_O-1}{i}\prod_{j=0}^{N_O-1-i}(1-\phi_j)\prod_{l=0}^{i}\phi_l \end{aligned} \tag{32}$$

The properties of this expression are not easily derived. Here we set out the derivative of p_i w.r.t. ϕ and we show that p_i decreases in N_O.

Consider first the effects of an increase in ϕ_k, i.e. an increase in the proportion of facets covered by firm k on the probability that firm i obtains a given facet. To investigate this we reexpress the probability of obtaining a facet as follows:

$$p_i = \prod_{j=1}^{N_O-1}(1-\phi_j) + \frac{N_O-1}{2}\cdot\phi_j\prod_{j=1}^{N_O-2}(1-\phi_j) + \frac{(N_O-2)(N_O-1)}{6}\prod_{j=1}^{N_O-3}(1-\phi_j)\prod_{j=N_O-3}^{N_O-1}(\phi_j)...\ ,$$

$$\begin{aligned}
&=(1-\phi_k)\prod_{j=1}^{N_O-2}(1-\phi_j)+\tfrac{1}{2}\phi_k\prod_{j=1}^{N_O-2}(1-\phi_j)+\frac{N_O-2}{2}\phi_j(1-\phi_k)\prod_{j=1}^{N_O-3}(1-\phi_j)\\
&\qquad+\frac{(N_O-2)}{3}\phi_k\phi_j\prod_{j=1}^{N_O-3}(1-\phi_j)+\frac{1}{3}\frac{(N_O-3)(N_O-2)}{2}(1-\phi_k)\phi_j\prod_{j=1}^{N_O-3}(1-\phi_j)\ \dots,\\
&=\left[\sum_{i=0}^{N_O-1}\left[(1-\phi_k)\frac{1}{i+1}+\phi_k\frac{1}{(i+2)}\right]\binom{N_O-2}{i}\prod_{j=0}^{N_O-2-i}(1-\phi_j)\prod_{j=0}^{i}\phi_j\right]+(1-\phi_k)\frac{1}{N_O-1}\phi_j^{N_O-2}
\end{aligned}\tag{33}$$

Then the derivative is:

$$\frac{\partial p_i}{\partial \phi_k}=\left[\sum_{i=0}^{N_O-1}\left[-\frac{1}{(i+1)(i+2)}\right]\binom{N_O-2}{i}\prod_{j=0}^{N_O-2-i}(1-\phi_j)\prod_{j=0}^{i}\phi_j\right]-\frac{1}{N_O-1}\phi_j^{N_O-2}<0 \quad . \tag{34}$$

Finally consider the effects of an increase in N_O on the probability of patenting a facet:

$$\begin{aligned}
p_i(N_O+1)-p_i(N_O)=&\sum_{i=0}^{N_O}\frac{1}{i+1}\binom{N_O}{i}\prod_{j=1}^{N_O-i}(1-\phi_j)\prod_{j=0}^{i}\phi_j\\
&\qquad-\sum_{i=0}^{N_O-1}\frac{1}{i+1}\binom{N_O-1}{i}\prod_{j=1}^{N_O-1-i}(1-\phi_j)\prod_{j=0}^{i}\phi_j\\
=&\left[\sum_{i=0}^{N_O-1}(-\phi_j)\frac{1}{i+1}\binom{N_O-1}{i}\prod_{j=1}^{N_O-1-i}(1-\phi_j)\prod_{j=0}^{i}\phi_j\right]+\frac{1}{N_O+1}\phi_j^{N_O}\leq 0
\end{aligned}\tag{35}$$

We also plot the function, allowing ϕ and N_O to vary.

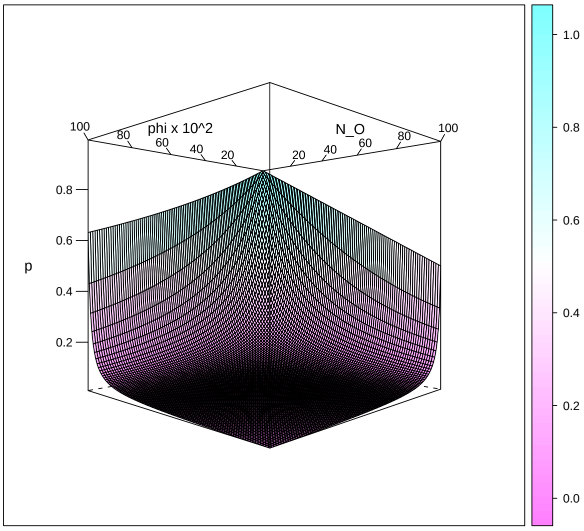

Figure 7: Simulation of p_i for $N_O \in 0, 100$ and $\phi \in 0, 1$.

B Robustness of the Empirical Model

Table 7: Patent Applications Estimates using OLS and Fixed Effects

	OLS models			Fixed effects models		
Variable	OLS 1	OLS 2	OLS 3	FE 1	FE 2	FE 3
log Patentcount$_{t-1}$	0.599***	0.583***	0.583***	0.172***	0.157***	0.156***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)
log Patentcount$_{t-1}$ × Triples		0.001***	0.001***		0.001***	0.001***
		(0.000)	(0.000)		(0.000)	(0.000)
Non Patent References (NPR)	0.064***	0.076***	0.067***	0.002	0.016	-0.007
	(0.002)	(0.002)	(0.003)	(0.007)	(0.008)	(0.009)
NPR × Triples		-0.002***	-0.002***		0.000	0.000
		(0.000)	(0.000)		(0.000)	(0.000)
NPR × Triples × Large			0.000*			0.000
			(0.000)			(0.000)
NPR × Large			0.020***			0.038***
			(0.004)			(0.006)
Fragmentation	29.910***	30.332***	30.352***	34.246***	33.811***	33.825***
	(0.269)	(0.320)	(0.320)	(0.346)	(0.392)	(0.392)
Fragmentation × Triples		-0.028***	-0.028***		0.016	0.016
		(0.007)	(0.007)		(0.009)	(0.009)
Triples	0.000***	0.002***	0.002***	0.001***	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Areas	0.018***	0.018***	0.018***	0.084***	0.084***	0.084***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Large	0.279***	0.282***	0.256***	0.305***	0.306***	0.263***
	(0.004)	(0.004)	(0.006)	(0.005)	(0.005)	(0.009)
Year dummies	YES	YES	YES	YES	YES	YES
Primary area dummies	YES	YES	YES	YES	YES	YES
Constant	0.122***	0.116***	0.128***	0.029	0.031*	0.060***
	(0.011)	(0.011)	(0.012)	(0.015)	(0.016)	(0.016)
R-squared	0.671	0.672	0.672	0.300	0.301	0.301
N	173448	173448	173448	173448	173448	173448

*$p<0.05$, ** $p<0.01$, *** $p<0.001$

C Complex and Discrete Technologies

Table 8: Classification of technology areas according to OST-INPI/FhG-ISI

Area Code	Description	Classification
1	Electrical machinery, electrical energy	Complex
2	Audiovisual technology	Complex
3	Telecommunications	Complex
4	Information technology	Complex
5	Semiconductors	Complex
6	Optics	Complex
7	Analysis, measurement, control technology	Complex
8	Medical technology	Complex
9	Nuclear engineering	Complex
10	Organic fine chemistry	Discrete
11	Macromolecular chemistry, polymers	Discrete
12	Pharmaceuticals, cosmetics	Discrete
13	Biotechnology	Discrete
14	Agriculture, food chemistry	Discrete
15	Chemical and petrol industry, basic materials chemistry	Discrete
16	Chemical engineering	Discrete
17	Surface technology, coating	Discrete
18	Materials, metallurgy	Discrete
19	Materials processing, textiles paper	Discrete
20	Handling, printing	Discrete
21	Agricultural and food processing, machinery and apparatus	Discrete
22	Environmental technology	Complex
23	Machine tools	Complex
24	Engines, pumps and turbines	Complex
25	Thermal processes and apparatus	Complex
26	Mechanical elements	Complex
27	Transport	Complex
28	Space technology, weapons	Complex
29	Consumer goods and equipments	Complex
30	Civil engineering, building, mining	Complex

Description of the 30 technology areas contained in the OST-INPI/FhG-ISI technology nomenclature. We classified the 30 technology areas as complex or discrete attempting to replicate the classification of Cohen et al. (2000).